STUDENT BEHAVIORS AND WAYS THEY ARE ASSOCIATED WITH THE PROBLEM SOLVING PROCESS

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We analyze data from classrooms to build upon pilot research in which we found an association between certain types of student behaviors and the development of understanding. The pilot focused on an after school program in a suburban setting. In this case, we focus on several inner city classrooms. Our findings indicate that there are many similarities. These results may have important implications for understanding how student behaviors evolve over the course of a particular problem solving experience.

Introduction and Theoretical Framework

Introduction: This research is one component of a larger longitudinal study in which University researchers partner with teachers in a large and densely populated urban school district to provide professional development (Schorr, Warner, Gearhart & Samuels, 2007). In the present study, we analyze data collected from two classrooms in an effort to build upon research gleaned from a pilot study that took place in a middle-class suburban after-school center (Warner, 2005). In the pilot, the behaviors of the students were examined as their mathematical understanding of a particular idea grew (using the Pirie/Kieren model for the growth of mathematical understanding). Results suggest that there appears to be a relationship between certain types of student behaviors and the growth of a particular strand of ideas. These behaviors include, for example, building and linking representations, reorganizing and questioning your own and/or others' ideas, setting up hypothetical situations, etc.. In the current research, we test these hypotheses in 2 other classroom situations. Our results suggest that such similarities do take place, along with a few differences. This has important implications for understanding how student behaviors evolve over the course of a particular problem solving experience.

Theoretical Framework: The Pirie-Kieren model for the growth of understanding (Pirie & Kieren, 1994) provides a framework for analyzing student growth in understanding, via a number of layers through which students move both forward and backward. Pirie (1988) discussed the idea of using categories in characterizing the growth of understanding, observing understanding as a whole dynamic process and not as a single or multi-valued acquisition, nor as a linear combination of knowledge categories. Pirie & Kieren (1994) illustrate eight potential layers or distinct modes within the growth of understanding for a specific person, on any specific topic: primitive knowing, image making, image having, property noticing, formalizing, observing, structuring and inventizing. In Warner (2005) a relationship between student behaviors and movement through layers in the Pirie-Kieren model was reported on. The results were based upon detailed case studies of three middle-school students' problem solving efforts as they solved a series of combinatorics tasks with the same or similar underlying structures. A summary of the observed behaviors and the associated layers in the Pirie-Kieren model follows:

> There was a general marked decrease in students re-explaining, questioning and/or using their own or others' ideas, with this being the most frequent behavior occurring when the students were working in the inner-layers or early stages of understanding.
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> Students moved to new representations at a fairly constant rate throughout the process, with instances of this behavior occurring in each of the layers.
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> There was a slight increase in students reorganizing or building on their own or others' ideas, beginning with a high frequency in the inner-layers of understanding.
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> There was a general marked increase in students linking representations to each other, with a low frequency in the inner-layers of understanding.
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> There was a marked increase in students setting up hypothetical situation and connecting contexts, with almost no instances of these behaviors in the inner layers.

These findings suggest that as understanding grows, there is a general shift in behaviors such as students questioning each other, toward those involving the setting up of hypothetical situations, linking of representations and connecting of contexts. The present study allows us to investigate whether or not these same associations occur in the context of two inner-city classroom settings. As part of the current study, we also revise and refine our associations to better account for the observed behaviors.

Methodology

Background: The research reported on in this study takes place in the largest urban school district in the state of New Jersey.

Data: As with the pilot study, the data that forms the basis for this study consists of at least six months of videotaped sessions (regular classroom sessions in the present study). In each case, at least two video cameras captured different views of the students' group work, students' presentations, etc. All student work was collected and descriptive field notes were compiled.

Analysis: In this study, the investigation of a task, over several classroom sessions, is analyzed using observations, field notes and videotapes. In both classrooms, we analyze the students investigating a combinatorics problem involving a mathematical structure of the form: $n(n - 1)/2$. The problem essentially is to find the fewest number of calls that needs to be made when 15 people call each other only once. It is also important to note that the students in each classroom explored a task with the same underlying structure (the "handshake problem", NCTM, 1989), at least 4 months prior to these sessions.

Selected episodes were identified based upon student(s) understanding in the outer-layers of the Pirie-Kieren model. We traced these moments back in time in an effort to document specific behaviors or actions on the part of students (see bullet points above) that appeared to contribute to movement through the layers of the Pirie-Kieren model. Detailed summaries and transcript were compiled.

Results and Discussion

Our analysis, which is thus far consistent with the pilot study, suggests that a number of student behaviors appear to be associated with the growth of mathematical ideas in specific ways. For example, in one particular case, which was typical of many others, an eighth grade student began her problem solving experience (for a task similar in structure to one that was used in the pilot) in the image-making layer (doing something to create an image). There were many instances in which she explained, used, and questioned her own and others' ideas. Others also questioned her. During the early stages of understanding, it is to be expected that students would in fact, question each other, and build representations that others don't understand at the time, thereby resulting in repeated questioning and requests for explanations and clarifications.

As this student moved from the image making layer to the image having layer (reaching a "don't need boundary" where she is no longer tied to the action), we began to see more instances of her moving to new representations and reorganizing and building upon other's representations. We suggest that one explanation for this is that she wanted to refine her representation in order to make it more useable to her, and in order to address the challenges of her peers. This is consistent with activity in this stage of understanding, which is still in the process of formation until the learner has an 'image.' In the process of moving from image having to property noticing (connecting her images to each other), we began to see more instances of her linking representations to each other and setting up hypothetical situations. These behaviors began to increase as she moved to, and began working in, the formalizing layer (creating a "for all" statement). As an example, she began to set up many hypothetical situations until she began to realize how the ideas she had been considering would correspond to any case, for any number. We suggest that when working in the formalizing layer, a student seeks to "tie up loose ends" as she links representations to each other in an effort to justify her formal statement.

In our findings, there wasn't a decrease in students re-explaining, questioning and/or using their own or others' ideas, as students moved to the outer-layers or later stages of understanding (which was the case with the pilot study). In both of the classrooms, we found that the teachers' actions may explain this inconsistency. For example, each teacher spent a good deal of time highlighting selected students' ideas and encouraging them to present their ideas to the class after they moved to the outer-layers of understanding. They also encouraged the students in the "audience" to ask questions. In one of these classrooms, in particular, the justification questions asked by students in the audience appeared to heavily influence the students' move to another outer-layer of understanding.

Conclusion

It is our hope that by documenting the relationship between these behaviors and the development of understanding, at least for this type of mathematical problem, we can shed more light on the problem solving process in general, and when one might anticipate the occurrence of specific behaviors as students are working on mathematical problems.

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