Responding To Students' Mathematical Thinking When You Don't Know How To Respond

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We share how several middle school teachers dealt with situations in which they did not understand their students' thinking. Our goal is to discuss strategies and decisions related to dealing with the complexities of classroom situations involving cognitively demanding tasks.

Purpose of the Session: We focus on the ways in which four urban middle school math teachers, each implementing the same task, dealt with situations in which they did not understand the mathematical ideas of their students. We share how each handled the situation, and the impact of their decisions on their students. Our goal is to openly discuss important strategies, decisions, and outcomes related to helping teachers become more comfortable with the complexities of classroom situations in which they allow students to solve cognitively demanding tasks, especially when confronted with confusing, unexpected or unanticipated student ideas. Investigating this collectively is of upmost importance, especially in light of the standards outlined in CCSS and NCTM documents (NGA and CCSSO, 2010; NCTM, 2014).

Research base: Stein, Smith, Henningsen & Silver (2009) emphasize the importance of providing students with opportunities to solve high cognitive demand tasks. Such tasks are often complex, and may involve multiple solutions, representations, and justifications; some of which may be unanticipated or not easily understood by teachers as they occur in real time. Stein, Engle, Smith, & Hughes (2008) underscore how difficult this can be for teachers: "Because solution paths are usually not specified for these kinds of tasks, students tend to approach them in unique and sometimes unanticipated ways" (pg. 313)…and facilitating a discussion around a task that can be solved in numerous ways greatly reduces teachers' degree of control over what is likely to happen in a lesson…"(pg. 323). Thus, teachers may be confronted with strategies or methods that are both difficult to understand and even harder to orchestrate as they unfold. How teachers respond to students as they solve such tasks greatly impacts when and how the students will learn the mathematical ideas involved (Jacob, Lamb and Philipp, 2010; Schoenfeld, 2011, Warner, Schorr and Warner, 2014).

Background: The research that is the basis for this presentation took place during a yearlong professional development (PD) project[1]. PD sessions occurred in person, online, and in schools. During these sessions, teachers investigated many ideas, including the implementation of high cognitive demand tasks. They investigated mathematical ideas, solution strategies, student learning trajectories, implementation plans, and other issues relevant to their classrooms. After implementing some of the tasks, they met with researchers to reflect on the process and review student work. In this session, we focus on the implementation of a task that involved finding the number of blocks needed to build towers of varying heights, and generalize to towers 'n' high.

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Examples to be shared during the presentation: Mr. R. tried to make sense of a student's idea, but noted that when he still couldn't understand her thinking, he suggested that she use the solution path that he had used when solving the problem. The result was that the student appeared to become discouraged. She complied, and abandoned her strategy. Mr. R reacted similarly to several other students. Our data suggest that the students did not understand why the teacher's solution strategy worked. At the end of the class, Mr. R noted that he did this because he realized that the students were having difficulties, but did not know how to deal with them. He felt safest in having them work on the strategy that *he* understood.

When confronted with a similar situation, Ms. J encouraged a student to present her idea to the class. She also suggested that the other students test the method to see if and when it worked. Much to her surprise, the method seemed to work. During the PD reflective session that followed, Ms. J noted that she was frustrated by her inability to understand this method during the class. It was not until she was able to review the idea with her peers in the onsite debriefing and PD session that she actually gained insight into the method.

Ms. F didn't understand a solution while a student was presenting it to the class (during group sharing). In response, Ms. F. told the student (in front of the class) that there was something wrong with the solution strategy. She then quickly ended the presentation so as to avoid any further frustration. She nonetheless praised the whole group, in the hope that the student would not walk away feeling bad; the rest of the class applauded.

Ms. I looked at a student's paper during group work. She realized that there was a solution that was incorrect, however, she wasn't sure why. Ms. I told the student to look at another student's paper (whose work she did understand). The student then erased his work and copied the other work.

During the PD sessions that followed, the teachers openly shared these and other implementation issues. Mr. R said that he "really felt bad as a teacher" because he "wasn't prepared as the instructor on how to guide it [the class discussion]". Ms. J said she was "frustrated" because she "still had no idea" how the student came up with the solution and was "scared" to give her students word problems because she may not "know how to respond" to a student, and then get "stuck." Other teachers discussed strategies for dealing with these types of situations. For example, they decided that, at the very least, they needed to explore additional ideas prior to implementation. They also noted the importance of accepting the fact that there were going to be times when they did not have an immediate answer for, or understanding of, everything all of the time. Project researchers had discussed this, but the teachers developed a deeper and more meaningful understanding of this when they saw that their peers, whom they respected, had the same or similar experiences.

Presentation Organization: Participants will be given an opportunity to consider the problem, and discuss both their own and projected student solution possibilities and (mis)understandings (15minutes). We will share the research and teacher examples above (15 minutes); participants will then be encouraged to discuss what they might do in similar situations (15 minutes).

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