Figure Captions

Figure 1. The Pirie-Kieren layers (Pirie & Kieren, 1994)

Figure 2. Folding back

Figure 3a. Anna’s tree with words for the Flag problem
Figure 3b. Anna reorganizes her tree with words for the Flag Problem
Figure 3c. Anna uses a five tree for the Flag Problem
Figure 3d. David’s tree with numbers tied to exponents for the Flag Problem
Figure 3e. David’s use of “n” to represent the number of extensions for the Flag Problem
Figure 3f. David’s use of X^y for the Flag Problem
Figure 3g. David’s use of Pascal’s Triangle for the Equivalent Trains Problem

Figure 4a. Anna’s tree with words for the Castle Problem
Figure 4b. Anna reorganizes her tree with words for the Castle Problem
Figure 4c. Anna’s revision for the Castle Problem
Figure 4d. Anna’s three tree for the Castle Problem
Figure 4e. David’s tree with words for the Castle Problem
Figure 4f. David reorganizes his tree with words for the Castle Problem
Figure 4g. David’s use of Pascal’s triangle for the Castle Problem
Figure 4h. David’s connection between the numbers in Pascal’s triangle and letters

Figure 5a. Michelina’s use of X^y for the Building Towers Problem
Figure 5b. Michelina’s attempt at an organized list for the Flag Problem
Figure 5c. Michelina’s attempt at a multiplication representation for Flags
Figure 5d. Michelina linking a tree representations to exponents for the Flag Problem
Figure 5e. Michelina switching back to an incorrect use of the base for Flags
Figure 5f. Michelina’s next attempt at linking a tree representation to exponents for Flags
Figure 6a. Michelina’s use of $X^y$ as soon as she read the Castle Problem
Figure 6b. Michelina sets up a hypothetical situation for castles and uses her $X^y$ to solve
Figure 6c. Michelina’s confusion about the base and exponent for the Castle Problem
Figure 6d. Michelina’s confusion about representing multiplication with a tree for Castles
Figure 6e. Michelina moving to a tree representation to explain $5^2$ without certainty
Figure 7a. Michelina links representations to each other for the Castle Problem
Figure 7b. Michelina’s explanation of the connection for the Castle Problem
Figure 8. Anna connecting formal statements to each other
Figure 9. David connects contexts
Figure 10. David links representations for castles
Figure 11. David links representations for flags
Figure 12a. Equivalent trains relating to other tasks
Figure 12b. Explanation of the tree representation for trains
Figure 12c. Pascal Triangle representation for trains
Figure 13. David folds back to the image making layer