INVESTIGATING STUDENTS' MOVEMENT TO GENERALIZATION AND ABSTRACTION USING DIENES' "LEARNING CYCLE"

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Introduction & Theoretical Framework: Generalization can be described as: "…deliberately extending the range of reasoning or communication beyond the case or cases considered, explicitly identifying and exposing commonality across cases, or lifting the reasoning or communication to a level where the focus is no longer on the cases or situation themselves but rather on the patterns, procedures, structures and the relationships across and among them" (Kaput, 1999, p. 136). Students' abilities to construct generalizations are often linked to their engagement in the process of abstraction (Davydov, 1990), which provides them with a means to move beyond the context of a given problem and transfer their knowledge to new problem situations. For the purposes of this study, it was useful to consider Dienes' notion of the 'learning cycle' (Dienes, 1970) in order to examine the ways in which students engage in generalization and abstraction during mathematics learning. According to the six stages within the "learning cycle", students first engage in a pattern of preliminary, unstructured exploration with a given concept, extend their learning through consequent, more structured explorations, and later, test the applicability of the concept in new situations before it becomes a 'functional' idea to them. Using Dienes' ideas as a lens, we document and examine students' ability to move beyond the mathematical context of a combinatorics task toward generalizations, which in this case, involves exponential, quadratic, cubic and other functions of that nature, and their graphs.

Methods: For this poster session, we analyze video-data of ten ninety-minute sessions in an eighth grade inner-city mathematics classroom, while students worked on a problem-solving task.

Preliminary Findings: One finding demonstrates that after students had found local solutions to a specific problem, a student-posed hypothetical situation (one that extended the problem beyond the specifics of the original task), highlighted by the teacher, contributed to students' movement to generalizations. The students found patterns as a result of unstructured exploration. For example, as the students were exploring exponential functions on the graphing calculator, they noticed that as their bases increased, the graph of the function remained in the first quadrant. One particular student posed the question about how to move their functions into other quadrants, which appeared to contribute to the students' exploration of other exponential functions, and later, functions with a variable base and constant exponent. Consequently, students developed formal statements to describe the behavior of their functions. Moreover, as time elapsed, the students began to pose more of their own hypothetical situations, thereby prompting themselves to extend their ideas through additional self-structured explorations, and ultimately, were able to use the new mathematical concept in a functional way.

References

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