Beliefs and Engagement Structures: Behind the Affective Dimension of Mathematical Learning

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Research on how students' beliefs influence their mathematical learning and problem solving suggests that beliefs themselves are structured, that they link to each other forming belief systems, and that they are intertwined or embedded in larger affective as well as cognitive structures. This theoretical article explores one sort of structure with which beliefs may be intertwined – a psychological notion termed an *engagement structure.* Engagement structures are idealized constructs hypothesized to help account for recurring patterns in complex affective and social interactions that occur "in the moment" as students work in groups on conceptually challenging mathematics. Analogous in some ways to cognitive structures, engagement structures can become active under specific conditions, with important immediate and longer-term consequences for children's learning of mathematics. Examples include those we call "Get The Job Done," "Look How Smart I Am," "Don't Disrespect Me," "Check This Out," "I'm Really Into This," and "Let Me Teach You." We present the idea of engagement structures in a self-contained way, and suggest how beliefs are characteristically woven into their fabric so as to influence their activation under particular circumstances. The research is based on a series of studies of middle school students in urban, inner-city classrooms in the United States.

Introduction and Background

The context of the research

In an urban middle school classroom in a low-income, predominantly minority community in the United States, early adolescent children are working in small groups on a conceptually challenging mathematics problem. Their teacher, having presented the problem, moves from group to group providing encouragement and asking occasional questions. When the teacher is not present, the children work with each other. Some are occupied individually, deeply engaged in the mathematics. Some are in conversation with others in their group, discussing the problem or talking about apparently unrelated things. Some appear distracted, uninterested, or bored. Others seem stuck or confused, and here and there occur expressions of frustration. One student tries to explain the problem to another who is having difficulty understanding it. Sometimes a student disagrees with or criticizes another's ideas, and occasionally a student takes offense at criticism. One or two groups have solved the problem, and a student expresses her disappointment that it turned out to be so easy.

As these and numerous other social and affective interactions take place, the students' mathematical understandings and problem solving strategies – generally speaking, their mathematical cognitions – also vary considerably. Individuals respond to the teacher's instructions in different ways. They have different prior knowledge, construct different interpretations of the problem, propose different representations, and adopt different strategic approaches. They bear many distinct and changing mathematical conceptions and misconceptions, and engage differently with the ideas or procedures proposed by other students. They grapple in a variety of ways with underlying mathematical structures in the problem that involve additive, multiplicative, or recursive processes.

The students have brought into the classroom situation a spectrum of individual beliefs – about the school setting and its expectations, about their peers and how they stand with their peers, about their teacher and their relationship to the teacher, about mathematics and what it means to learn or do mathematics, about parental expectations, about their own mathematical capabilities and interests, and so forth. In complex ways, the beliefs of the individual students interact with and influence their social interactions, their mathematical problem solving, and the nature of their engagement with the mathematical task at hand.

In such an environment, how can one *understand usefully* the many, complex influences on the students' cognitive, affective, and/or behavioral engagement with mathematics – the *why* of the dynamics of their "in the moment" engagement? Taking student interest and engagement to be a key affordance of meaningful learning, how can one identify and study teaching strategies that result in increased engagement – not only in the particular socioeconomic setting being studied, but more generally across different venues?

This article introduces and explores the theoretical notion of *engagement structures*, a psychological construct developed by the authors to help account for recurring, dynamical patterns observed in affective and social interactions as students work in groups on conceptually challenging mathematics. Briefly, an engagement structure is an idealization that involves a characteristic motivating desire and associated goals, implementation actions toward fulfilling the motivating desire, supporting beliefs, "self-talk," sequences of emotional states, meta-affect, modes of interaction with mathematical tasks, and possible outcomes. Thus it is a kind of *behavioral/affective/social constellation* that is situated in the individual as a psychological construct, becoming active in social contexts. Examples discussed below include those we call "Get The Job Done (GTJD)," "Look How Smart I Am (LHSIA)," "Don't Disrespect Me (DDM)," "Check This Out (CTO)," "I'm Really Into This (IRIT)," "Let Me Teach You (LMTY)," "It's Not Fair (INF), "Stay Out of Trouble (SOOT)," and "Pseudo-Engagement (PE)."

Our goal here is to present the underlying ideas in a self-contained way, to discuss their connection with other constructs in the literature on affect and motivation, and to explore, in a preliminary fashion, the intimate relationship between engagement structures and the beliefs that are characteristically woven into their fabric. We thus develop a language with which to discuss mathematical engagement with teachers and researchers – particularly the affective dimension of engagement and learning, as it manifests itself in classroom interactions. We conjecture that *the activation of particular engagement structures is an important mechanism whereby individual students' beliefs influence their "in the moment" interaction with the mathematical task at hand.*

In the remainder of this introductory section, we mention some of the theoretical ideas influencing our work: ideas about affect and beliefs in the context of mathematical learning, and ideas about motivation and engagement in educational contexts. We seek to situate our "engagement structures" construct in the wider context of this research.

Affect, beliefs, and mathematical learning

The affective domain as it pertains to mathematics teaching, learning, and problem solving has been described as including emotional feelings, attitudes, beliefs, and values. McLeod (1992, 1994) discusses emotions, attitudes, beliefs, and relationships among them. He takes emotions to be rapidly changing (momentary or transitory) and highly affective, attitudes to be more stable and incorporating more elements from the cognitive domain, and beliefs to be the most enduring component and understood to be highly cognitive as well as affective. DeBellis & Goldin (1997, 1999, 2006) suggest considering values (including ethics and morals) as a distinct affective component, while Schoenfeld (2010) makes use of the broader term “orientations” to include “beliefs, values, biases, dispositions, etc.” (p. viii).

Evans (2000) addresses emotional aspects of adult learners of mathematics, and Malmivuori (2001) offers in-depth discussion of affect and the social environment in relation to mathematical learning; for further perspectives, see Gomez-Chacon (2000a,b); Hannula (2002, 2004); Lesh, Hamilton, & Kaput (2007); Zan, Brown, Evans, & Hannula (2006); Malmivuori (2006); Evans, Morgan, & Tsatsaroni (2006); and the references therein. Pioneering work in the psychology of emotion has focused on relations between affect and cognition (e.g., Csikszentmihalyi, 1992; Dai & Sternberg, 2004), and self-identity or beliefs and theories about the self (Steele & Aronson, 1995; Dweck, 2000).

Goldin (2000) discusses affect as a *system of internal representation* in the individual, encoding information and interacting with verbal, imagistic, and strategic representational systems during problem solving; this is a perspective that we also take here. Of particular importance to the current study is also the considerable body of research on affective issues in relation to social environments for children in schools within low-income, minority urban communities (e.g. Anderson, 1999, Dance, 2002).

From these and other sources, we understand the affective domain to be both *individual* and *social*, to interact continuously with cognition, to function as a system that represents, encodes, and communicates information, to have immediate “in the moment” consequences for mathematical learning, and to have these “local” effects in ways that essentially involve longer-term, more “global” structures that include beliefs.

The book edited by Leder, Pehkonen, and Tőrner (2002) offers a variety of different perspectives on beliefs, all which emphasize their central importance to mathematics education. Op ‘t Eynde, De Corte, and Verschaffel (2002) distinguish among students' beliefs about mathematics education, their beliefs about themselves, and their beliefs about the social context in which they are learning. McLeod and McLeod (2002) review some of the important consequences of mathematical beliefs and open questions about them, from early work on the limitations they can impose during mathematical problem solving (e.g. Schoenfeld, 1985) to differences among researchers in their very definition. Goldin (2002) advocates a definition of beliefs as "multiply-encoded, internal cognitive/affective configurations, to which the holder attributes truth value of some kind (e.g., empirical truth, validity, or applicability)" (p. 59). An important aspect of affect, particularly in relation to beliefs, is the concept of *meta-affect*, which includes "affect about affect, affect about and within cognition that may again be about affect, monitoring of affect, and affect as monitoring." (p. 59); Goldin suggests that "prevailing belief structures ... are powerfully stabilized by meta-affect. Such beliefs are unlikely to change simply because factual warrants for alternate beliefs are offered." (p. 70). Likewise, beliefs can stabilize meta-affect. For further discussion, see DeBellis & Goldin (1997, 1999, 2006), and Goldin (2007).

Of particular interest is research addressing the complex interactions among emotions, beliefs, and the educational contexts in which they occur. Pekrun, Frenzel, Goetz, and Perry (2007), for example, consider "emotions tied directly to achievement activities or achievement outcomes," offering a taxonomy of such achievement emotions. They provide an overview of the "control-value theory" that predicts how patterns of appraisals can result in the occurrence of different achievement emotions. They note that "control-related beliefs (e.g., self-concepts of ability) and value-related beliefs (e.g., individual interests) can be assumed to affect appraisals and resulting achievement emotions ... For example, if a student holds favorable control beliefs regarding her achievement in an academic domain like mathematics, an activation of these beliefs will lead to appraisals of challenging tasks as being manageable, and to related positive emotions." (p. 25).

Schoenfeld (2010) develops his theoretical approach to teachers' "in the moment" decision-making based on the three components of goals, resources, and orientations. Articles in the book edited by Maass and Schlöglmann (2009) elaborate on the structured nature of beliefs generally, and beliefs pertaining to mathematics and mathematical learning in particular. Here Goldin, Rösken, and Törner (2009) discuss the notion that beliefs themselves are structured (cf. Törner, 2002), that they link to each other forming belief systems (cf. Green, 1971), and that they are embedded in larger affective as well as cognitive structures (Goldin, Epstein, & Schorr, 2007). It is certain of these *affective structures* – i.e., the engagement structures – and their relation to beliefs that are the major topic of the present article.

Motivation and engagement

In the psychology of personality, it is often helpful to distinguish *traits* from *states* – the former referring generally to longer-term, stable characteristics of the individual; the latter referring to the more rapidly changing particulars that influence behavior "in the moment" (Cattell & Scheier, 1961). In mathematics education, much of the study of the affective domain – even the study of emotional feelings, such as anxiety – has tended to focus on the more trait-like characteristics. *Attitudes* can be understood either as propensities toward certain kinds of behavior, or propensities toward certain kinds of emotional feelings in relation to mathematics; they vary from person to person, but are thought to change relatively slowly. Thus researchers have made use of instruments such as the Mathematics Anxiety Rating Scale (Richardson & Suinn, 1972; see also Capraro, Capraro, & Henson, 2001), the Mathematics Attitudes Scales of Fennema and Sherman (1976), and similar measures. Typically, positive emotional feelings or attitudes have been taken as desirable, negative ones as undesirable.

Likewise *beliefs*, particularly "control" and "value" beliefs as mentioned above, are seen as "trait-like student characteristics that in interaction with the classroom context are thought to influence [mathematical problem solving] processes" (Op 't Eynde, De Corte, & Verschaffel, 2007, p. 191). Students can be classified into "types" based on the positivity of their belief profiles, according to Op 't Eynde *et al.*'s Mathematics-Related Beliefs Questionnaire. Students' beliefs are shown to be closely related to the emotional feelings and perceptions they report during mathematical problem solving in the classroom, seen as a product of cognitive, affective, and conative processes.

However, the characterization of affect (including attitudes and beliefs) as "positive" or "negative" can conceal some important ambiguities, complexities, and essential features. *Meta-affect* can transform affect, so that negative feelings (such as fear, anxiety, frustration, or anger) can all, under certain conditions, be experienced positively. Patterns of affect associated with constructive engagement do not exclusively involve "positive" emotional feelings such as curiosity, excitement, fun, or satisfaction; but inevitably include feelings of impasse, frustration, and disappointment as well. When the emotional journey resulting in mathematical success has been arduous for the student, the resulting sense of satisfaction and achievement may be more profound and longer-lasting. As mathematics educators, we must come to understand how "negative" feelings also can support mathematical engagement, persistence, and learning. Likewise, "negative" beliefs about mathematics or about oneself in relation to mathematics can contribute under some conditions to powerful and constructive behaviors – for example, the belief that mathematics is abstract and difficult, and that one's own ability to do it is limited, can sometimes engender deep determination, hard work, and persistence.

There is much evidence for the existence of a positive relationship between student engagement and academic achievement (Finn, 1993; Greenwood, 1991; Marks, 2000). Park (2005) consistently finds positive effects for student engagement, regardless of SES or minority status, on students' mathematical growth. To carry out such studies, "engagement" must be understood as more than an "in the moment" state; there must typically be a way to attribute different degrees of engagement to different students over time, pointing in the direction of "trait-like" (i.e., steady or slowly changing) characteristics. Features of individuals' motivations, such as their achievement goal orientations, may be likewise understood as "trait-like" (e.g., Schutz & Pekrun, 2007; Schunk & Zimmerman, 2008; and references therein). These include "mastery-approach goals" focusing on achieving (mathematical) understanding of the material, as opposed to "performance-approach goals" focusing on achieving as much as or more than others. Linnenbrink (2007) proposes "... a triarchic model of reciprocal causations ... in which there are reciprocal relations among achievement goal orientations and affect, affect and engagement, and achievement goal orientations and engagement" (p. 122).

Another important distinction in the literature on motivation is between "intrinsic" and "extrinsic" *rewards*, the former being associated with the "informing function" of reward and the latter with the "controlling function" (Zimmerman & Schunk, 2008).

The concept of engagement structures, described in greater detail below, is intended to assist in understanding the *dynamics* of "in the moment" engagement by individual students, in interaction with others, solving conceptually challenging mathematical problems. In analogy with cognitive structures, we hypothesize that most or all of the engagement structures we describe develop in most or all individuals. Engagement structures interact with and draw upon beliefs. Different engagement structures can become active, in the same individual, under different social conditions, and can call upon others or branch into others as the situation evolves. That is, they exist "long term" in individuals, but they are *not* the sort of "traits" intended to distinguish individual students from each other (like belief systems, motivational orientations, or personality characteristics). Discussing the activation of engagement structures helps us to describe and understand the complexity of students' "states" in classroom situations.

Indeed, we hope to be able to identify certain engagement structures as more likely to be activated in association with (for example) mastery-approach goals, positive beliefs or attitudes, and/or intrinsic motivation; and others more likely to be activated in association with (for example) performance-approach goals, negative or mixed beliefs, and/or extrinsic motivation. One way this can happen is for the trait-like features to help direct the person's attention preferentially toward certain features of the environment, influencing their selection. The selected environmental features then play a key role in activating a given structure. Thus, two individuals with different traits in similar environments have different potentials for activating particular engagement structures.

We also hope to be able to identify teaching strategies that activate (or help to construct) various engagement structures appropriate to the learning occasion. Achieving optimal mathematical engagement is then a matter of creating a classroom environment in which this is accomplished.

In the next section, we describe briefly the empirical studies that led to our developing the notion of engagement structures.

Empirical studies and methods

Initial exploratory study

We set out during 2006-07 to implement an *exploratory* study of affect in urban, middle-school mathematics classrooms. Our initial framework was eclectic, drawing on ideas from mathematics education, cognitive science, urban studies, and social psychology. We sought to investigate, using qualitative methods, some broad research questions, including: How do teachers create an *emotionally safe* classroom environment for urban students to engage in *conceptually challenging* mathematics? What contributes to (or impedes) the students' development of powerful mathematical affect? What are the cognitive and affective consequences for the children, including their social interactions, emotional states, and mathematical learning? How do these develop over the school year?

Conceptually challenging mathematical activity involves gaining or changing some understanding, and usually entails some experience of impasse during nonroutine thinking or problem solving. Typically some change of representation is needed, or some new representation must be constructed. Mathematical meanings are at least as important as procedures. Classroom activity may include exploration, discussion, and argumentation, with individual students expressing their own ideas and conjectures. Wrong answers and blind alleys occur, as well as fruitful suggestions. In this context, students are likely to challenge and criticize each others' ideas. Such challenges are affective as well as cognitive, and may evoke strong emotional feelings – leading sometimes to deeper engagement with the mathematics, or at other times to disengagement.

An *emotionally safe environment* is taken to be one where the social interactions during mathematical inquiry (which can include mistakes, false starts, criticism of each others' ideas, and impasse) do not entail experiences of fear, pain, humiliation and shame, or domination and submission. Rather, the children's normative experiences include trust, confidence, dignity, and shared respect in doing mathematics (Schorr & Goldin, 2008).

Our initial study focused on classrooms of three teachers known to encourage mathematical exploration and discussion, and thought to be skilled in creating emotionally safe classroom environments. The main underlying conjecture was (and remains) that developing *powerful affect* in relation to conceptually challenging mathematics is fundamental to the growth of mathematical ability, and essential to mathematical achievement. Here *powerful affect* refers to structured patterns of emotional feelings and affective representation, including beliefs, that foster children's intimate engagement, interest, concentration, and persistence to the point of mathematical success.

In each of the three classrooms, the research team collected data during four or five "cycles" over the school year. Four "focus students" in each class were selected in advance to reflect a cross-section by gender and apparent degree of emotional expressiveness. Two videographers with roving cameras followed the focus students in the classroom during each lesson, while a stationary camera captured an overview. Each cycle included a pre-interview with the teacher and videotaping of two consecutive lessons. Just after the second lesson in each cycle, the research team reviewed the videotapes and selected several evocative segments as a basis for individual, follow-up "stimulated recall" interviews with each focus student and with the teacher. At the beginning and end of the school year, additional pre- and post-interviews were conducted with each teacher, and specially-designed attitude surveys were administered to the experimental classes and to other classes in the same schools. Our intent was to capture the classroom activity and social interactions, including students' mathematical thinking and affective responses, as closely as possible (Davis & Maher, 1997).

Classroom and interview videos were transcribed and analyzed in a preliminary way through four "lenses." Our goal was to create coherent narratives describing: (1) the flow and development of mathematical ideas (cognitive/mathematical lens); (2) key affective events, by which we mean episodes where strong feeling or emotion is expressed or inferred, including the immediately subsequent development (affective lens); (3) social interactions among the students (social lens); and (4) significant interventions by the teacher (teacher intervention lens). Some of our methods and initial, qualitative empirical findings have been reported elsewhere (Alston, Goldin, Jones, McCulloch, Rossman, & Schmeelk, 2007; Epstein, Schorr, Goldin, Warner, Arias, Sanchez, Dunn, & Cain, 2007; Schorr, Warner, & Arias, 2008), and are not recapitulated in detail here.

The need for a new construct

In seeking to interpret the information gathered – particularly, the comparisons between episodes that occurred during mathematics class and the retrospective, stimulated-recall interviews with individual children – we concluded that analysis with respect to the four "lenses" with which we began was not adequate to understand our observations. A new theoretical construct was needed to characterize, in an idealized way, the complex interactions of social, behavioral, and emotional aspects of the situations that we observed. We identified characteristic patterns of behavior and emotional feelings, confirmed by retrospective accounts, and these patterns seemed to recur, being evidenced by different students on different occasions. This fit with the idea that identifiable "affective structures" were being activated in particular social-environmental situations. At this stage of our work we hypothesized several such structures, inferring them initially from the videotaped classroom observations, and confirming where possible from the retrospective interviews. As we named specific structures and elaborated their description, it became considerably easier for us to discuss and interpret the events that had been videotaped.

We referred to these structures in earlier articles as *archetypal affective structures*, but here we adopt the less technical term *engagement structures*.

Continuing empirical study

We approached the next phase of our empirical work with the goal of exploring and validating, where possible, not only the overall concept of engagement structures but the specific structures that we had identified to that point. In this study, conducted during 2008-09, students in a set of middle school classrooms in a large urban school district worked in groups of three on a specific mathematical task – an exploratory, nonroutine activity in which the goal was to find an (algebraic) pattern or rule. The same task was used with all the different teachers, providing a partial control for task effects.

In place of roving cameras, we made use of a fixed videocamera at each table, with an additional audio pickup for clarity. Episodes in each group were analyzed with respect to behavior – verbal and nonverbal – that could be interpreted as evidence for each hypothesized engagement structure. In addition a questionnaire was designed (see below), administered after class, asking the students about the thoughts, feelings, and experiences that occurred for them during the class period. Each engagement structure that we had hypothesized to that point motivated specific questions. Then we were able to compare our interpretations of the students' behaviors with their answers to pertinent questions about their thoughts and feelings. We asked if our characterizations were consistent, for individual students, across their videotaped behaviors working in groups and their questionnaire answers after the class period, with respect to inferences we were making about engagement structures that had been active.

We next widened the study to school districts with different socioeconomic characteristics, and to classroom activities involving a variety of mathematical tasks. We found some possible relationships between (1) measures relating to engagement structures, and (2) socioeconomic context, teachers' professional development experiences, and the choice of problem task (Schorr, Epstein, Goldin, Warner, & Arias, in preparation). As the numbers of participating students increased, we were also able to obtain information through factor analysis and other techniques about the clustering of students' responses to various questionnaire items (Epstein, Goldin, Schorr, Capraro, Capraro, & Warner, 2010).

Our findings were encouraging with regard to the validity of the engagement structure concept and most of the particular structures we described; and they formed the basis for further modifications of the theory. Nevertheless, we regard all that we are proposing here as theoretical and tentative, with larger-scale experimental confirmation – and, most importantly, exploration of the conditions that influence the development and activation of engagement structures – to follow. To advance this program, a new questionnaire instrument based explicitly on the activation of engagement structures is presently under development.

In the next section, we present a general but more detailed description of engagement structures, including several specific examples. We then discuss a number of their theoretical aspects. The section that follows is devoted to hypothesized relationships between beliefs and engagement structures, elaborating on the structured nature of individuals' beliefs and their interface with the social dimension. The article concludes with some implications for mathematics education and future research.

Engagement structures

General description

Engagement structures are psychological constructs that we hypothesize to be present in individuals, and to become active (i.e., to influence or govern behavior, thoughts and feelings) in particular social-environmental situations. They are highly affective, and recurrent. We distinguish them from a wider class of "affective structures" by noting that their features are oriented toward a particular *motivating desire* – an "in the moment" goal of the individual.

Let us consider, first, the parallel strands that, woven together, constitute such a structure when it is fully developed. These are to be regarded as components that are *simultaneously occurring* and *mutually interacting*. It is the nature of the distinct strands that leads us to characterize an engagement structure as being a "behavioral/affective/social constellation." We identify and propose the importance of ten such strands (cf. Goldin, Epstein, & Schorr, 2007):

(1) A characteristic *goal* or *motivating desire*, evoked by particular circumstances in the social environment. We relate the motivating desire for each engagement structure to the individual's sensing an opportunity to fulfill a manifest or latent *need* in the sense discussed by Henry Murray in his classic book, *Explorations in Personality* (Murray, 2008, 70th Anniversary Edition). Motivating desires are much more concrete and situation-specific than needs. Our perspective is that in the (classroom-based) social situation where the student is working with others on a conceptually-challenging mathematics problem, the perceived opportunity arises to satisfy a need by pursuing some more concrete, immediate goal. The *environmental press* (Murray's term for situational constraints facing the person) interacts with the individual's need, impelling *actions taken* toward satisfying the motivating desire.
(2) A characteristic *pattern of behavior*, beginning in response to the particular circumstances in the social environment evoking the motivating desire, and oriented toward fulfilling the desire. Characteristic behavioral outcomes and contingencies may form part of the pattern.
(3) A characteristic sequence of emotional feelings, or *affective pathway*, experienced (internally) by the individual.
(4) *Expressions from which affect may be inferred* that are socioculturally-dependent as well as idiosyncratic, which can also serve some communicative function. These include emotionally expressive words, eye contact and facial expressions, posture and "body language," hand and body movements including touching or making gestures toward others, interjections and exclamations, agitation, tears and laughter, blushing, etc.
(5) Information or *meanings encoded by the emotional feelings*.
(6) *Meta-affect*, that includes feelings about feelings, feelings about cognition about feelings, and self-monitoring of affect.

(7) Characteristic *self-talk* or inner speech, in response to and evocative of the person's emotional feelings, beliefs, and underlying motivating desire. The concept of self-statements emanates from research in cognitive therapy for emotional disorders (Beck, 1976; Hollon & Beck, 1994).
(8) Interactions with the individual's *systems of beliefs and values* – these are a central focus of the current discussion, addressed in greater detail below.
(9) Interactions with the individual's longer-term structures of *self-identity, integrity, intimacy* or other affective structures, personality traits, and *motivational orientations*.
(10) Characteristic *problem-solving strategies and heuristics* for decision-making.

Clearly some of these components have themselves been objects of considerable research attention in the mathematics education community, while others have not. In our work, we now take the student's experience of a specific goal or motivating desire (as expressed at the time, or in a retrospective interview, or in response to a questionnaire), *together with* evidence of behavior toward achieving the goal or fulfilling the motivating desire (as observed at the time, or reported subsequently), as the minimum for inferring activation of a particular engagement structure during the class.

The adjective "archetypal," which we used in our earlier research, is intended to suggest two things: first, the *idealized* nature of our descriptions of these structures; and second, the *universality* or near-universality of the presence in individuals of the structures we describe. We stress that *archetypes are not stereotypes.* We are *not* interpreting engagement structures as describing stereotypical behavior of a social or cultural group, but as describing patterns to be found in individuals in most or all human groups, cultures, and contexts.

Engagement structures *develop* as the child grows. The actual psychological structures in individuals are, of course, likely to differ from our idealized descriptions. But the theoretical structures are intended to provide both a *language* for discussion (with teachers as well as researchers), and a *template* for the interpretation of observations.

Examples of engagement structures

We next describe some specific examples of engagement structures that we have identified or conjectured from observations of middle school students working in groups in mathematics classes. We are presently exploring 14 distinct structures; we mention here 9 of these, for which we have the most extensive or compelling preliminary evidence; see also Schorr, Epstein, Warner, & Arias (2010a,b).
. For each structure, we highlight the motivating desire, the need (in the sense of Murray) that the motivating desire may be addressing, features of the social situation likely to evoke the motivating desire, and some of the consequent behavior and/or emotional feelings. We are then ready to discuss, in the subsection that follows, some additional aspects of the "engagement structures" construct, including its relation to beliefs.

(1) "Get The Job Done" (GTJD). The student's motivating desire in this engagement structure is to satisfy a sense of obligation to complete an assigned mathematical task, to correctly follow the instructions that are given, or to meet a commitment. Underlying this goal may be the need Murray calls *deference:* "to yield to the influence of an allied other" [the teacher] (p. 154). The desire is typically evoked by the teacher's directions to the class. Emotional satisfaction follows ultimately from having fulfilled the commitment, not necessarily from having achieved an understanding of the mathematics. The consequent behavior is oriented toward efficient or straightforward completion of the assigned activity. In a team or group context, the student may seek to enlist others in accomplishing the task. This is one of the most commonly observed structures in mathematics classrooms.

(2) "Look How Smart I Am" (LHSIA). The student's motivating desire here is to impress others (or, possibly, himself or herself) with the student's mathematical ability, knowledge, intelligence, or genius. Behind this desire may be the need Murray terms *achievement:* "to increase self-regard by the exercise of talent" (p. 164). The desire may be evoked by a potentially admiring audience, or possibly the presence of "rivals" for achieving high regard. The consequent behavior can be competitive, including "showing off" the student's prowess by trying to demonstrate that his or her solution is better than that of others. Emotional

satisfaction accompanies the achievement of recognition, if it occurs, that the student's own thinking or achievement is superior.

(3) "Check This Out" (CTO). In this engagement structure, the motivating desire comes from the student's realization that solving the mathematical problem can have a "payoff" or benefit – immediately, or in the future – that the student wants. The payoff may be intrinsic (e.g., a possible use or application of the mathematics), but it is not always so. The need behind this desire may vary with the nature of the benefit. The desire is evoked situationally by perception of the payoff possibility, and the goal is to achieve the payoff. The consequent behavior is increased attention to the task in pursuit of it. Depending on the nature of the payoff, the result can be increased (intrinsic) interest in the task itself, or heightened (extrinsic) interest in that an association has been established between the mathematics and the (non-mathematical) payoff.

(4) "I'm Really Into This" (IRIT). Here the student's motivating desire is to experience the very activity of addressing the task, having the experience of "flow" (Csikszentmihalyi, 1992). The student is intrigued by the mathematics or the processes of problem solving to the point of "tuning out" other elements of the environment. Behind this desire, in the case of mathematics, may be the need Murray calls *understanding:* "to represent in symbols the order of nature" (p. 224). Situationally, the opportunity presents itself in the social environmental support for deep engagement in a challenging problem. Satisfaction or a sense of accomplishment may be derived from achieving a full mathematical understanding, from solving a difficult problem, or simply from the experience of fascination during profound active involvement.

(5) "Don't Disrespect Me" (DDM). In this engagement structure, the motivating desire is to meet a perceived challenge or threat to the student's dignity, status, or sense of self-respect and well-being. The likely underlying need is termed by Murray *infravoidance:* "to avoid conditions which may lead to belittlement" (p. 192). Typically the social context is that of a challenge to the student's expression of a mathematical idea, where the challenge is perceived as belittling or insulting. The consequent resistance to the challenge – defending oneself – raises the

conflict to a level above that of the original task. The need to "save face" then can override the issue of understanding mathematical concepts, for instance in the context of a highly-charged discussion or argument.

(6) "Stay Out Of Trouble" (SOOT). The motivating desire underlying this structure is to achieve safety, avoid interactions that may lead to conflict or emotional distress (e.g., embarrassment, humiliation, or anger) involving either peers or someone in authority. Murray describes the need for *harmavoidance:* "to take precautionary measures" (p. 197). The social context suggests to the student the possibility of being punished, embarrassed, humiliated, or otherwise hurt by others. Avoidance behavior follows, as aversion to risk supersedes addressing the task's mathematical content. A sense of relief rewards success in avoiding the potentially troublesome situations.

(7) "It's Not Fair" (INF). In this engagement structure, the motivating desire is to redress a perceived inequity. The underlying need may be what Murray terms *succorance:* "to have one's needs gratified by an allied other" (p. 182). The desire is evoked as the student experiences some unfairness in a group problem-solving effort; e.g., with the level of participation by others in the group, or with the role accorded to the student. This leads to a disinvestment in the mathematical ideas in the task, and an investment in corrective behavior to restore fairness or balance to the situation. Satisfaction, if it occurs, derives from achieving such a restoration, or if not, just "getting it done and over with."

(8) "Let Me Teach You" (LMTY). Here the motivating desire is to help another student understand or to show him or her how to solve the mathematical problem. Included in the need that Murray identifies as *nurturance* is: "to gratify the needs of a mentally confused person" (p. 184). The social situation evocative of the motivating desire is one in which the student becomes aware of someone who does not understand or is confused, while the student has a mathematical insight or relevant knowledge that can be shared. The consequent behavior is to try to assist another, to demonstrate a method or explain a concept. Satisfaction is derived from the other student learning and/or appreciating the help.

(9) “Pseudo-Engagement” (PE). In this structure, the motivating desire is to look good (to the teacher, or to peers) by appearing to be engaged with the mathematical task, while avoiding genuine participation in problem solving activity. The underlying need is termed by Murray *blame avoidance:* “to avoid blame or rejection” (p. 187). The desire arises in situations where genuine participation is not perceived by the student as possible or satisfying, while others (the teacher or fellow-students) might potentially blame or punish overt disengagement. Consequent behavior may include trying to look busy, or other actions that present an image of engagement. Relief or reduction of tension occurs as the mathematical activity comes to an end without the student’s detachment from it having been noticed.

Further discussion

Stages in active engagement structures

For each engagement structure, we developed an idealized description of the stages that occur as it plays itself out successfully in a classroom context, focusing on emotional feelings and self-talk. These stages are: (A) the *initial activation* of the structure, the accompanying emotional feelings, self-talk, and mathematical activity; (B) the *initial behaviors* toward achieving the motivating desire, the accompanying emotions, self-talk, and strategy for fulfilling the desire; (C) the *continuation* of the engagement structure, including (successful) implementation of the strategy, emotional feelings, self-talk, and mathematical activity, and (D) the *outcome*, (ideally) the achievement of the object of the motivating desire, accompanying emotional feelings, and consequences for mathematical learning.

We made use of our description of these stages to develop the preliminary questionnaire instrument that we used in the second phase of our empirical study, mentioned in the subsection above entitled “Continuation Study.”

Specificity and universality of engagement structures

Engagement structures are not the only kinds of affective structures – other affective structures described in the literature include self-identity and self-efficacy, integrity, and so forth. We have characterized engagement structures by their alignment around fulfilling a particular motivating desire. Likewise, many different kinds of motivating desires arise in the everyday lives of middle school children. Our focus on contexts where the students are engaged in conceptually challenging mathematics leads us here to identify those engagement structures inferred from observations in mathematics classrooms. Some of these structures no doubt become active in other contexts, too – "Don't Disrespect Me," "It's Not Fair," "Get The Job Done," and so forth. But we also expect that, in other contexts, other structures not discussed here would come importantly into play.

We noted above our perspective that engagement structures are not specific to any one social context, and are not specific to any particular cultural, racial, or ethnic groups. To take one example, some of the face-saving issues central to the "Don't Disrespect Me" structure have been described persuasively in studies of inner-city street life (Anderson, 1999; Dance, 2002). But we understand that the same engagement structure can be inferred from behavior in college faculty meetings, in situations involving difficult negotiations, in formal social gatherings, and in many everyday contexts (as well as in school classrooms engaged in mathematical discussions). The particular *expressions* of the affect – the fourth component in our list of ten mutually interacting strands above – can differ substantially according to different sociocultural norms in different contexts, as well as across individuals; but the underlying affective *structure* remains essentially invariant.

Desirability of engagement structures

We stress also that *we do not regard some of these structures as "good" and others as "bad."* Rather, we see most or all of them as universally present in individuals, and each serves important functions – regulating affect, cognition, and social behavior. The challenge for the teacher is to create an environment for addressing conceptually challenging mathematics that is *emotionally safe* – so that (for example) serious mathematical engagement with integrity (with active structures ranging from "Get The Job Done" to "I'm Really Into This" to "Look How Smart I Am") contributes to (rather than jeopardizes) safety, status and "face," and is experienced as leading away from "trouble" rather than toward it.

Branch points in engagement structures

Let us pursue somewhat further the analogy between engagement structures and the perhaps more familiar idea of cognitive structures. During the course of problem solving, the solver may pursue a number of different heuristic processes or strategies. As this plays out, the solver's preestablished cognitive structures or schemata are likely be accessed; i.e., to become active. For example, a governing strategy of systematic trial and error, in a situation where two constraints are imposed in a mathematical problem, may entail making use of or "activating" simultaneous coordination of conditions. As the solver does this, there may occur an opportunity to draw on (or "activate") proportionality or proportional reasoning (e.g., if at least one of the conditions being coordinated is multiplicative). But as the problem solving proceeds, the solver might possibly notice a path toward solution that makes use of proportional reasoning without further trials, and abandon the trial and error and coordination of conditions. One might describe this process as *branching* from one active cognitive structure to a different one. A structure initially accessed as a kind of "subroutine" becomes the governing one.

Similarly, as we study classrooms where students are working in groups on mathematical problems, we sometimes notice what seem to be critical "choice points" or "branch points" in engagement structures. As we refer to them here, such branch points occur when someone can act (consciously or otherwise) in such a way as to *change the motivating desire*, thus activating a different structure and experiencing a different set of thoughts and feelings. Particularly when events do not unfold in the expected or hoped-for way (e.g., due to the nature of classmates' responses or the teacher's interventions), one engagement structure may become inactive, and another may come into play.

For example, in our work we have seen many instances where a peer challenges a student's work. This can elicit a series of actions typical of the "Don't Disrespect Me" structure, with the student immediately becoming defensive of her position – often to the point of unwillingness to actually consider the argument of the other student. Subsequent comments are interpreted as "attacks" on her mathematical identity. As the student engages in the defense of her ideas, however, she may come to feel sufficiently secure that she begins to take seriously the comments or arguments of the student who challenged her. If something in those comments suggests a possible payoff, for instance by offering a different perspective on the problem, her subsequent responses may be more consistent with the "Check This Out" engagement structure, with "Don't Disrespect Me" no longer active.

Likewise, a student may set out to "Get The Job Done," but notice along the way that he understands something another student does not. Initially, "Let Me Teach You" becomes active in service of the original motivating desire of having the group complete the assigned task. As he becomes more engaged with the teaching of his fellow-student, the imparting of understanding may become the major motivating desire, with the goal of simply completing the original task no longer salient; GTJD has branched into LMTY.

Alternatively, as one student attempts to teach another, it may develop that his peer does not regard him as especially knowledgeable or smart, and is not prepared to accept the help. Then the engagement structure LMTY may branch into "Look How Smart I Am," as he tries to impress his peer with his knowledge or ability. The latter, accessed initially in service of the motivating desire LMTY, may rapidly become the governing engagement structure.

In short these affective structures, developed within each individual, not only call upon each other but can supersede each other in the course of the changing social situation.

Beliefs intertwined with and acting through engagement structures

Now we arrive at an important point in our discussion – the hypothesized relationship of individuals' beliefs and belief structures, and their associated values, to engagement structures. Recall that we regard engagement structures as archetypal, with most or all of them present in most or all individuals; and their activation is descriptive of the individual's *state.* Beliefs and belief structures are particular to individuals, and (except for the most transient beliefs) for part of the description of the individual's *traits*.

We take beliefs as propositions or imagery held by the person as true or valid; the beliefs may also be warranted through reasoning and evaluation of evidence or experience, so as to involve the person's cognitions. Furthermore, beliefs are normally supported by emotional feelings and meta-affect – they may meet emotional needs, and provide (for example) defense from pain. We have noted research that supports the relationship of "control" and "value" beliefs to motivational orientations, as well as to emotional feelings experienced during problem solving. Here we hypothesize that *beliefs are intertwined with engagement structures, so as to influence their activation under specific environmental and social conditions.* This suggests a specific mechanism through which beliefs influence students' "in the moment" mathematical engagement.

Let us consider some specific, highly idealized beliefs or structures of beliefs, in order to explore how this can work.

One such structure involves a student's belief that mathematical ability is inborn and innate. Failure or mediocre performance in mathematics is therefore "not my fault." In fact, the student takes a certain amount of pride in declaring that "I am just not a math person – I wasn't born with it." The belief is a part of the student's self-identity in relation to mathematics. Related beliefs may include the ideas that success in mathematical tasks is mainly a matter of knowing the right computational or problem solving procedures, that high ability consists of being able to remember complex procedures easily and perform them rapidly and accurately, that high ability shows up, therefore, in high test scores, and that the student herself or himself cannot (therefore) normally expect a high grade. This belief structure may function to assuage guilt, providing the student under some conditions with good reasons to disengage before frustration can arise.

Under ordinary classroom conditions, these beliefs may support activation of engagement structures such as "Get The Job Done," "Stay Out Of Trouble," or "Pseudo-Engagement," while impeding activation of structures such as "I'm Really Into This," "Look How Smart I Am," or "Let Me Teach You." For instance, GTJD allows the student holding these beliefs to bring to bear the knowledge that he does have, to complete the task expeditiously and procedurally if possible, to ask for some step-by-step help from the teacher or another student, and to detach from further engagement with the mathematics *without calling the beliefs into question.* In contrast, IRIT – should it occur – might threaten some or all of the beliefs, forcing an emotionally unwelcome change in the student's mathematical identity. Thus, the belief structure facilitates activation of certain structures, and impedes others, when the student faces a conceptually challenging mathematics problem in class in a group situation.

This does *not* mean that this particular student lacks the IRIT engagement structure. It may well become active in other social situations, ranging from playing sports to participating in a school play – just not in the context of conceptually challenging mathematics.

As remarked above, beliefs can directly faciliate the activation of engagement structures by influencing the selection of particular environmental features for attention. Consider, for example, a female student holding the belief that teachers think girls are not good at math. When her teacher calls on a boy instead of her, she attends to this and atrributes it to what she believes to be the teacher's belief. This increases the likelihood of activating the "It's Not Fair" structure – particularly, if the girl thinks she has a good answer. She is motivated in class subsequently by her desire to rectify the perceived injustice. Alternatively, she may branch into "Get The Job Done" as a way to comply with instructions while disinvesting in further conceptual learning.

A student who did not hold her belief would be less likely, under similar circumstances, to select this aspect of the teacher's behavior for attention. Similarly, an African-American student who believes that teachers favor white or Hispanic students might under parallel conditions be more inclined toward the activation of INF than a student without this belief.

These examples suggest that for each of the engagement structures introduced above, one may identify commonly-held mathematical beliefs or systems of belief, and related values, which – while generally speaking not *necessary* to the activation of the structure – could at least theoretically facilitate it under some circumstances:

GTJD beliefs and values: mathematics as procedural, answer-oriented, and rule-governed, requiring thoroughness, with the teacher as the authority in setting tasks; compliance and meeting expectations are valued;

LHSIA beliefs and values: mathematics as a domain that requires high innate ability or genius; high self-efficacy; mathematical ability is highly valued;

CTO beliefs and values: mathematics as having some internal logic and/or some valuable areas of application; sufficient self-efficacy to achieve the perceived payoff by working at the problem; the payoff and possibly conscientious work are valued;

IRIT beliefs and values: mathematics, mathematical representation, or problem solving as intriguing, having internal logic and coherence; self-concept or self-identity as being an effective problem solver, engaged thinker, or serious student; problem-solving or learning activity is highly valued for its own sake;

DDM beliefs and values: mathematical correctness of answers or reasoning as being important to status; self-concept as being capable of assertiveness and deserving of respect; maintaining status is highly valued;

SOOT beliefs and values: mathematics or mathematical problem solving as dangerous or strewn with pitfalls; low ability to defend oneself if challenged, or low emotional or intellectual self-efficacy; conflict avoidance is highly valued

INF beliefs and values: school mathematical activity as entailing implicit rules of fairness in division of work; existence of bias in recognition of individuals' or groups' abilities and contributions to problem solving; equality of treatment and sharing fairly are highly valued;

LMTY beliefs and values: mathematics as having some internal logic; high self-efficacy; mathematics is valued as something worth understanding; helping someone else is likewise valued;

PE beliefs and values: mathematics as difficult, unpleasant, boring, and/or inaccessible; low self-efficacy or possibly unwarranted high self-concept; satisfactory opinions of others, or avoidance of negative opinions, are valued.

Further research and potential implications

The ideas set forth in this article, while empirically based, are still at a relatively early stage of development. We have embarked on further research to validate, if possible, and extend, where appropriate, the concepts we have described, and to confirm or disconfirm the hypotheses we have suggested – the particular engagement structures described, and their relationship with beliefs and belief systems. In particular, the development of new questionnaire instruments based on engagement structures permits the systematic test of specific relationships between those structures active during mathematics classes, and students' mathematical beliefs and achievement orientations.

Much of the study of affect and motivation in mathematics education has until now tended to emphasize trait-like characteristics of teachers and students, and their relation to longer-term outcomes. We propose that important and fruitful results may ensue from the study of students' individual and classroom mathematical behavior through the lens of engagement structures. As we come to identify the most important such structures and characterize them more precisely, we are learning what to take as persuasive evidence that a particular structure is present and functioning. We must also learn how to recognize and influence the choices students make at the most critical branch points. And we hope to learn – and be able to document – teaching strategies that foster engagement in conceptually challenging mathematics.

In short, we suggest that the "engagement structures" construct holds promise as a possible key to understanding "in the moment" emotion and engagement in mathematics, and how these are influenced by beliefs. As we improve our observational techniques, the construct allows us to examine some of the affective consequences of various interventions. And developing the language that permits us to consider engagement at this "structural" level facilitates professional discussions with mathematics teachers and the growth of explicit awareness, as teachers recognize many of the social/behavioral patterns described. Our work to this point in inner-city classrooms suggests the importance and value of such a research program.

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