Working with a Partner:
An Investigation of Student Engagement in an Middle School Math Classroom

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Abstract

Many students work in small groups as they solve mathematical problems. In order for them to benefit from such experiences, each individual must be engaged, at least to some extent, during the problem solving session. Such engagement may take many forms, and in this paper, we focus on three closely related types of engagement, and discuss them with respect to both cognition and affect. We provide classroom examples of each, including student explanations of their mathematical ideas and reasoning. In addition, we provide qualitative descriptions of quantitative data as gathered in response to questionnaire items.

Objective

The purpose of this research is to analyze some of the different types of interactions that take place as small groups of students work together to solve mathematical problems. The students in this study attended an urban middle-school in which they were provided with many opportunities to solve complex problems individually and in groups. Several studies on small-group work in mathematics classes focus on the outcomes or performance of the students as a consequence of their peer-to-peer interactions (e.g., Cohen, 1994; Webb, 1991; Webb & Mastergeorge, 2003). These studies tend to focus on conditions that lead to more productive group work, such as group configuration (e.g., heterogeneous/homogeneous) (e.g., Hooper & Hannifin, 1988; Swing & Peterson, 1982), the nature of the interactions (e.g., what skills assisted with productive group work) (e.g., Cobb, Yackel, & Wood, 1992; Webb, 1982, 1991), and communication of mathematical ideas (e.g., Webb, 1991; Weber, Maher, Powell, & Lee, 2008). By "productive groups," researchers may refer either to achievement outcomes measured by tests or also to "development of higher order thinking skills" (Cohen, 1994, p. 3). Our research and analysis in this study allows us to explore students' mathematical ideas and reasoning during these interactions from three different, yet intertwined perspectives: social, cognitive, and affective, as students explain concepts, procedures and other details to one another.

Theoretical Framework

We explore the interactions between and amongst students working in small groups in conjunction with each student's emotional and cognitive engagement. Specifically, we consider three of the many different ways in which students engage with one another as they work through a mathematics problem (described below). Using brief episodes from our classroom data, we present three engagement structures (referred to in some of our previous work as affective structures), based upon the work of Goldin, Epstein, & Schorr (2007), Epstein et al. (2007), and Schorr, Epstein, Warner & Arias (2010-a,b). The concept of **engagement structure** refers to an idealized, recurring highly affective pattern inferred from observed behavior. It is a *behavioral/affective/social constellation* which may become active in a given social context.

Each engagement structure includes, "a characteristic motivating desire, one or more goals, implementation actions to achieve the motivating desire (sometimes involving social interactions and sometimes involving other patterns of behavior), "self-talk" (which refers to hypothetical internal speech), sequences of emotional states, strategies, and modes of interaction with mathematical tasks" (Schorr, Epstein, Goldin, Warner, & Arias, submitted for publication).

Schorr, Epstein, Goldin, Warner, and Arias (submitted for publication) make note of a critical aspect of engagement structures referred to as a **motivating desire**, meaning the "individual's desire, interest, sense of goal or purpose, inspiration, or aspiration to engage in or persist in an activity (see also Alderman, 2008; Eccles, Wigfield, & Schiefele, 1998; Schunk & Zimmerman, 2009)" (p. 2). The concept of the motivating desire is associated with a *need* as described by Murray (2008) in his book Explorations in Personality (70th Anniversary Edition). Murray's work examined the concept of personality, focusing on a number of variables including that of *need* and *press*. An individual's *needs* may not be observable but may nonetheless prompt the individual to act in a certain way to satisfy that need. The individual's actions may also be influenced by *press*, or a stimulus situation which is part of the environment and "usually appears in the guise of *threat of harm* or *promise of benefit*" (p. 41). In a classroom situation, a student's motivating desire may be evoked by a particular set of circumstances in the social environment, such as a mathematics class (Goldin, Epstein, & Schorr, 2007).

In this paper, we focus on three of the many different ways in which students may work together during a mathematical problem solving session. These three structures are related in that all involve situations in which a student is providing his or her classmate(s) with some kind of information about the mathematical problem or situation. However, the hypothesized motivating desires and resulting actions for each of the three structures are different. The structures are referred to as: "Let Me Teach You," "Look How Smart I Am," and "I'm Right, You're Wrong."

Let Me Teach You (LMTY) occurs when one student experiences a motivating desire to teach another person (tutee) something that he knows that the other person does not appear to know. The need underlying this motivating desire was described as *nurturance* by Murray (2008). In this situation, the student attempts to help his or her classmate understand the assigned problem or the mathematics, perhaps in the service of satisfying a need of the classmate. Ideally, the student would be successful in communicating the ideas to his classmate and would feel a sense of satisfaction from helping someone. However, it is also possible that the tutee may resist the help for some reason. As an illustration, consider the situation in which one student attempts to help another student who seems to be struggling with a mathematics problem. In one case, the second student may be appreciative of the help and may ask questions to further his own understanding of the problem and its solution. LMTY would then continue to be active for the tutor. In another potential case, the would-be tutee could be feeling frustrated because he is having difficulty understanding why his strategy is not working, or what the tutor is saying. Upon hearing his classmate offer some assistance or explanation, the second student may adopt a negative attitude toward the tutor and be unreceptive to suggestions or offers for help. The tutor, in that event, may not remain in the LMTY structure (at least with the tutee during that session). Indeed, he may even experience unpleasant feelings such as unhappiness, disappointment, etc. (Epstein et al, 2010), which in turn may evoke a different engagement structure for the would-be tutor.

Look How Smart I Am (LHSIA) can occur when a student hypothetically says to herself something like, "I know this but the others in my group do not," and subsequently realizes that this may be an opportunity to let the others see how much she does know. The need described by

Murray (2008) which we identified as underlying the motivating desire is referred to as *achievement,* specifically by boosting one's own intellectual status. The motivating desire is to impress others or "show off," and to have others recognize his or her mathematical ability, knowledge or intelligence. Such an individual may be assertive when expressing these ideas, as he or she may not be satisfied until others acknowledge how smart she is or how much she knows. Reactions by classmates may include acknowledging how much that student knows ("You're so smart), but may also involve others either ignoring or rejecting that student's contribution. If the student is either ignored or rejected, she may become defensive, either of her ideas or of herself. She may feel disrespected, and another engagement structure may be activated for this student.

I'm Right, You're Wrong (IRYW) may occur when one student believes his answer or strategy is correct and a classmate (or classmates) has an incorrect answer or strategy. The motivating desire behind this structure is to show others that the student is right and that another person is wrong. There may also be a desire to have others agree that "my way of solving a problem is better than yours." This motivating desire may be caused by the need Murray (2008) describes as, *dominance*, or "convincing others the rightness of one's opinions (p. 152). The student for whom IRYW is active may work to satisfy this desire by arguing in support of his ideas. For example, consider the situation in which the student's goal is to have others recognize that his answer or strategy is correct. Some ways in which others may respond to this student include recognizing that his idea is correct (regardless of whether the student's answer is indeed correct), asking questions to understand why that student believes his idea is correct, or rejecting the other student's ideas. If a classmate(s) respond in the second possible way, by asking questions, it is possible a LMTY structure may become active. If a classmate(s), instead, reject the student's ideas, that student may experience negative feelings toward his classmates.

The engagement structures LHSIA, IRYW, and LMTY may provide the opportunity for members of the group to learn something from their classmate. However, in the case of LMTY, a student is primarily motivated by the desire to have others learn and understand a mathematical idea or concept. In contrast, for LHSIA, the motivating desire involves how others perceive a student (i.e., as being smart). Similarly, for IRYW the student is motivated to demonstrate that he is correct and another student is wrong. Though a student for whom either LHSIA or IRYW is active may be attempting to share valuable knowledge that can help others, the way in which they present their information may discourage others from accepting any assistance. Because this student is primarily acting in service of himself instead of others, classmates may not benefit from the possible knowledge the student is attempting to impart.

We recognize the possibility that a student may experience one motivating desire and act in order to fulfill that need. However, the subsequent events may evoke a different motivating desire in that student, and another engagement structure may be activated in service of the second motivating desire. Schorr, Epstein, and Goldin suggest that one engagement structure may *branch* into another structure (Schorr, Epstein, Goldin, Warner & Arias, submitted for publication; Schorr, Epstein, Warner, & Arias, 2010-b). "Branch points typically correspond to outcomes of actions taken to satisfy the motivating desire" (Epstein et al. 2010). Several examples were given above. For instance, the student for whom LMTY was active may have experienced negative feelings because his offer to help another was rejected. At the point he realized his classmate was declining any assistance, the student may have *branched* into a different engagement structure (see Epstein et al., 2010 for a more complete list of structures). In addition, the student who was either looking for validation that she is smart or attempting to

show that another student was incorrect may have branched into LMTY if classmates started asking questions about the problem. Branch points may occur when attempts to satisfy the initial motivating desire are thwarted or when a classmate's response to the student's actions initiate a shift away from the original motivating desire.

To help us gain insight into the students' affect, engagement and motivating desires, the researchers of the larger project (see Epstein, Goldin, Schorr, Capraro, Capraro, & Warner, 2010; Schorr, Epstein, Goldin, Warner, & Arias, 2010-a, b) designed a retrospective questionnaire which was intended to measure the different ways in which the students were engaged during a lesson. Questionnaire items, described in more detail below, asked students about their motivating desires associated with each of the structures, including the two described in this paper. The questionnaire also attempted to assess possible actions taken by the students and possible outcomes, all related to the engagement structures. Some results involving the questionnaire and qualitative descriptions are described below.

Methods

Background: This study builds on a previous study in middle school mathematics classes in which the authors of this paper and other project researchers (Alston et al., 2008; Epstein et al., 2007 for details) investigated the following questions: "How can we explain how the actions, interactions, and statements of the students we are observing make sense? Why would they say what they said and do what they did" (see also Schorr, Epstein, Warner & Arias, 2010b). Answering these questions led to the development of the engagement structures by senior study researchers. The questionnaire, mentioned above and described below, was designed by the research team and piloted in a participating teacher's class in Spring 2008. A revised questionnaire was implemented during this study which took place between November 2008 and February 2009. After analysis of the questionnaire items and responses, the research team continued to revise the questionnaire and administered it to additional several hundred middle school students whose teachers were participating in professional development projects with university faculty and researchers (for more details see Epstein et al., 2010; Schorr, Goldin, Epstein, Warner, & Arias, submitted for publication).

The questionnaire administered to the students included in this analysis consisted of several types of questions: (1) 5 open-ended questions asking students to share memorable moments; (2) 42 items describing student thoughts or experiences during class using a 3-point Likert scale: 0 (never), 1 (some of the time), or 2 (all of the time); (3) a 22-item list of emotional feelings a student might experience list (11 positive and 11 negative) also using a 3-point Likert scale: 0 (not at all); 1 (somewhat), or 2 (very much); (4) 32 statements about the student's behaviors in class on a 3-point Likert scale: 0 (hardly ever), 1 (sometimes), or 2 (often), and (5) 25 yes/no items suggesting whether the student did or did not have a corresponding thought about the class. Examples of items which correlate with the structures LMTY, LHSIA, or IRYW are included in the Results.

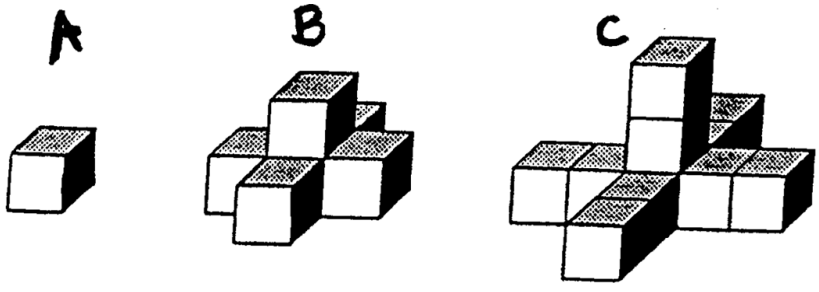
Subjects: The students highlighted in this study attended middle schools in a large urban school district in New Jersey where the student population is predominantly high poverty (87% of students eligible for free or reduced-priced lunch). A large number of students are from minority populations (27% African-American and 71% Hispanic). Four seventh- and eighth-grade teachers, each from a different middle school in the district, were invited to participate in the larger study based upon their openness to allowing students to work on challenging problems

over time and in groups and their desire to create a classroom culture that allowed students share solutions and offer explanations (as documented by project researchers over a period of at least two years). More specifically, the teachers generally provided students with opportunities to defend, argue, and justify their mathematical ideas on a regular basis (as documented through video-based data, personal reflections, and participation in professional development with project researchers). All four teachers had worked with senior university faculty and researchers as part of a long-term professional development project designed to encourage an emotionally safe classroom environment in which students could explore mathematical ideas and not be afraid to take risks (for more details see Schorr, Warner, Gearhart, & Samuels, 2007). For each teacher, data was gathered from two separate classes of students, for a total of eight classrooms.

All students in each class were placed into groups of three or four, using the following procedure. For each class, the students' names were first placed in alphabetical order by last name in an Excel worksheet. In the next column, the Excel function "RAND" was utilized to randomly generate a number between 0 and 1, associating each random number with a student. The column of randomly generated numbers was sorted from smallest to largest, rearranging the list of students' names accordingly. From that list, groups were created by placing the first three students together in one group, then the second three, and so on until all students were assigned to a group. A group of four was created if one or two students were excluded from a group of three, following the same order on the list. Once in the classroom, several groups were rearranged because students either were absent, no longer in the class, or had brought in their signed permission slip to participate in the study. In such cases, decisions to reorganize the groups were made on the spot, while attempting to keep the integrity of the original grouping.

For this study, students worked in small groups on a task originally selected by one of the study teachers in a pilot study[1]; this task was used in all study classrooms. The following task was designed to be conceptually challenging to most of the participating students. Students worked on the Building Blocks task in their small groups each day of observation. At the end of the observation cycle, students presented their work to their classmates and the teacher. Students completed the questionnaire individually each day. The figure below was included with the task, as well.

> *Building Block Task*: I was constructing towers as you see below. I noticed that each time I made the tower higher, I had to add more blocks on the sides to stabilize the structure. I would like to know how many cubes I will need to build a 5-block high tower and a 10-block high tower. Generalize, if you can, on how many blocks I will need for any size tower?



[1] Exemplars K-12 (2004), http://www.exemplars.com/resources/alignments/impact_course01.html

Analysis: We focus on two small groups of students here. One group consists of three seventh-grade female students in Ms. S's class. The group from Ms. B's eighth-grade class includes one female and two male students (gender selection was random). Each class was observed for two or three consecutive days during the normal class session in the fall of the school year (November and December 2008). Each small group was recorded via video and audio, and the student work was collected at the end of each day's session. In the last few minutes of the class period, students were asked to respond to the questionnaire described earlier. Our analysis is based on the first day of observation in each class.

We selected these two groups for our analysis after viewing several videos looking for qualitative evidence regarding student engagement structures in general. After repeated viewing, the research team concluded that that either LMTY and/or LHSIA may have been active for one or more students in the groups. Questionnaire responses for all students in the selected groups were then reviewed to support or disconfirm our hypothesis. We then reviewed the entire video looking for specific episodes demonstrating an active engagement structure. Upon reviewing the episodes, particularly those we highlight below, we recognized that the IRYW structure also appeared to be active for one of our students of interest. We revisited the questionnaire responses to again support or disconfirm our hypothesis. Portions of these episodes are highlighted below. In the next section, we share classroom episodes and present our analysis focusing on the engagement structures, cognition, and affect.

Results

In this section we present one episode from each of the two focus groups for this paper. First, we analyze the engagement and affect of the seventh-grade students in Ms. B's class: K (male), D (male), and R (female). Then we do the same for the eighth-grade students in Ms. S's class: T (female), A (female), and C (female). As mentioned above, once we inferred the Let Me Teach You (LMTY), Look How Smart I Am (LHSIA), or I'm Right, You're Wrong (IRYW) structure qualitatively from the video, we also viewed the questionnaire responses for the students in these two groups. Together, the findings from the video, the transcript, and the questionnaire may provide evidence that either one or both of the structures were active for any of the participating students. Both the tone of voice and significant gestures are also included in the episode transcripts, as they contribute to our inference of an active engagement structure.

Group I, Class I

In this first episode, which takes place about 30 minutes into the first problem-solving session, K and R were trying to show D the correct total number of blocks in a ten-block high tower. Earlier, D believed that the pattern was to multiply five times the height, which gave him an incorrect answer of twenty-five blocks for a tower five blocks high. His group and his teacher Ms. B had helped him to see that for a five-block high tower, there were a total of twenty-one blocks because there are five blocks for the height of the tower and only four blocks on each side. The students were then working to find to the total number of blocks in a ten-block high tower. K had already created a table (Figure 1), showing the total number of blocks for several consecutive heights, including five and ten. The analysis is intertwined throughout the continuous episode and focuses primarily on K.

At this point, D and R are each working with cubes, and K is watching them and listening.

D: It's still gonna be fifty though. [D seems slightly defensive and yet confident. He picks up the ten-block high tower structure that he built.] It's still fifty. [D is suggesting that the total number of blocks should equal fifty.]
K: No, it's not. [K's tone is matter-of-fact, as he looks over to D. K speaks softly, but is loud enough to be heard by those in his group and on the audio recordings.]
R: Forty-six! [R emphasizes her point – the correct total number of blocks in a ten-block high tower – by lightly banging her right hand against her desk twice.] Let me see it. [R reaches to take the tower out of D's hands.]
D: For that, it's still fifty. I fixed it already. [D's tone is defensive. He is referring to the tower structure he created.]
K: For that… [inaudible]… take one off. [K stands up and reaches across his and R's desks. He starts to take one block off each leg of the structure. R stops him. K remains standing.]
R: No, no, keep it. [R argues against K's action by taking the cube out of K's hand and putting it back on the structure.] He's right! [R is referring to D and his tower structure.]
K: No, he ain't. It's 50 here. [K is speaking slightly louder. K seems to believe that the block structure erroneously contains 50 blocks.] It's fifty here, but it's supposed to be forty-six. [K emphasizes "forty-six" by saying those words slightly drawn out and by lightly banging his right hand against the desk twice. He then sits down.]

There appears to be evidence here that K may have been motivated by the desire to show that his way was better than one of his classmate's ways for the following reasons. He tried to take the blocks off the structure even though R was holding it, indicating he could support his idea by creating the correct tower structure. He appeared to argue in support of his idea that there were forty-six blocks in the ten-block high structure by telling D that fifty blocks was incorrect with a matter-of-fact tone of voice, and by emphasizing this point when he stood up and when he struck the desk with his hand as he was saying, "forty-six." Together, the motivating desire (his way was better) and his action (attempting to "fix" the block structure and arguing for his idea) lead us to suggest that the IRYW structure may have been active. In Table 1, we notice that K responded *all the time* to the following questionnaire items. Specifically, K gave this response (*all the time*) for the motivating desire item, "I wanted to show someone that my way was better" and the action item, "I argued in support of my ideas."

Table 1: Questionnaire Items Which May Indicate I'm Right You're Wrong structure

Questionnaire Items - Statements	***K (m)**
I wanted to show someone that my way was better. (motivating desire)	All the time
I argued strongly in support of my ideas. (implementation action)	All the time
My ideas were challenged by others (potential outcome).	All the time
I liked to be right.	Often
Thoughts (Yes/No)	
I want you to admit you were wrong and I was right.	No

Response choices included: a) For statements: 0 never; 1 some of the time; 2 all of the time; b) For thoughts: yes; no; (c) For feelings: 0 not at all; 1 somewhat; 2 very much

Table 2: Questionnaire Items Which May Indicate Look How Smart I Am structure

Questionnaire Items – Statements	***K (m)**
I wanted people to think that I'm smart. (motivating desire)	All the time
I wanted the teacher to think that I am a good student. (motivating desire)	All the time
I tried to impress people with my ideas about the problem. (implementation action)	All the time
I felt smart.	All the time
People seemed impressed with the ideas I shared about the problem. (potential outcome)	All the time
People saw how good I was at the math today. (potential outcome)	All the time
I was a lot better at the math than others today.	All the time
Thoughts (Yes/No)	
I want you to know just how smart I am.	Yes
I wanted to show off.	Sometimes
I wish the teacher would call on me, so I can show how much I know.	Yes
Some Associated Feelings	
Respected	Very much
Proud	Very much
Successful	Very much
Satisfied	Very much
Confident	Very much
*Smart	Very Much

*Student wrote in this response

Next, R attempted to verify either D's or K's answers by counting the blocks in the existing tower structure. D started to write information down on his paper, in a table similar to the one K made. D asked K for the answers, which K provided.

D: How much is 2 [two-block high tower]? Six? [D starts writing information down on his paper.]

K: Yeah. [K is leaning over his desk towards D. K continues to quietly tell him the numbers for the total number of blocks, reading off his table. His focus jumps from his paper to D as he reads. D continues to write on his paper.] Twenty-one, twenty-six, thirty-one, thirty-six, forty-one.

R: This one is forty-six. This one is forty-six right here. [R has finished counting the blocks in the tower structure. Her tone of voice indicates satisfaction and has a, "Ha, told you so" ring to it.]

In response, K looks at R and returns to his normal seating position.

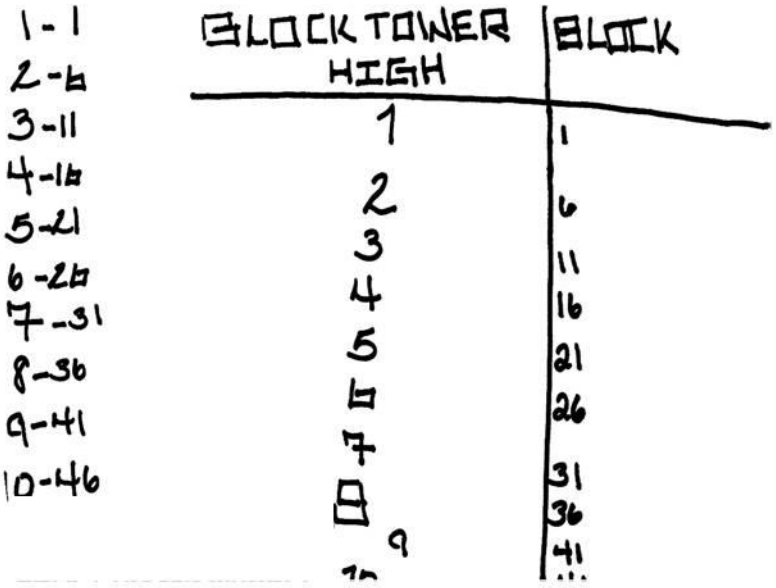


Figure 1: K's table

K could have had one of many motivating desires when he gave D the numbers to put in his table. He may have been motivated by the desire to show that he was smart (associated with LHSIA), or by the desire to show that he was right and D was wrong (associated with IRYW). K also may have been motivated to teach D something (associated with LMTY), though he did not provide any explanation for these answers. We recognize that still another desire may have motivated K and alternate engagement structure may have been active. Based on this episode thus far and the video of this group as a whole, we hypothesize that both LHSIA and IRYW structures were active for K. It might be the case that an IRYW structure was active as a means to support a LHSIA structure. Our questionnaire data shows that K responded *all the time* on the item: "I wanted people to think that I'm smart." Also, he responded *no* to the thought item: "I want you to admit you were wrong and I was right." Therefore, we infer that the IRYW structure was active based on K's observable behaviors, but perhaps he was more concerned with being smart than others being wrong. We therefore suggest that LHSIA was also active for K during the interaction above because of his questionnaire responses and K's actions throughout the episode - providing answers to his classmate.

When K provided D with the numbers for the table, he spoke softer than usual, perhaps so only D would hear him. K had already created his table and may have felt ownership over his answers. Throughout this episode, K expressed confidence that his answers were correct, and he may have felt a sense of satisfaction of being able to share those answers with someone. Indeed, K responded *very much* to questionnaire items asking if he felt "satisfied" or "confident" during the problem-solving session. We do not suggest that these feelings are exclusive to LHSIA, IRYW, or any individual structure, but we do believe the feelings are associated with these two engagement structures, particularly the LHSIA structure.

As the episode continues, even though R used the tower structure to demonstrate that there were forty-six blocks in the ten-block high tower, D was still confused. He asked, "Why

forty-six?" and did not get an answer from either K or R. D continued to argue that there were fifty blocks in this tower.

D: So, it's ten, twenty, thirty, forty, fifty.
[Both K and R lean in, to be closer to the structure. K is gesturing toward the legs of the tower.]
R: It's not…
D: It's ten on the side, right?
R: You don't count that one in the middle block five times.
[While R is speaking, K takes the structure away from D and puts it on his desk. While D and R have their exchange below, K counts and adds all the blocks in the structure. K never looks up at R and D during their exchange.]
D: Why not? [D's tone of voice is defiant]
R: Because you don't. [R and D are arguing here]
D: Yes you do.
R: No, you don't.
D: Okay, let's ask. [D's tone is conciliatory. He turns around to see where the teacher is.]
R: It's only one block. [She leans forward and puts up one finger.] So why would you count it five times? [R's tone indicates she is defensive of her ideas and is reproaching D for being incorrect.]
D: Because it counts. Without that middle piece, you got … [inaudible]
[D and R turn away from the conversation, perhaps waiting for the teacher Ms. B to settle their argument.]
K: Yup, it's forty-six. [K's tone of voice is confident and satisfied. He is still holding the tower structure and pointing to different parts with his hands. He glances at R.] I just took one off each side, like I said. [Note that K did not remove or add any blocks while he was counting those in the structure.]

When K makes his statement "It's forty-six," he is referring to the total number of blocks needed in a ten-block high tower. He states this with more emotion in his voice than in previous utterances, as though he feels confidence and pride, perhaps because he realized that his previous statements were justified after counting the blocks himself. Even though R had previously agreed with K, D had expressed doubt that forty-six was the correct answer. K may have been motivated to continue showing his answer was correct; if was able to prove he was correct, then he would be deemed 'smart' by his group. We infer that K expressed a feeling of success, from his tone of voice. This feeling may also correlate to his response of *often* to the questionnaire item, "I liked to be right."

Though K is not speaking for much of this portion of the episode, his actions speak for him. When counting the blocks, it seems K proved to himself that there were forty-six blocks in the ten-block high tower, as he stated repeatedly beforehand. This realization may have contributed to K feeling smart, a questionnaire item ("I felt smart") to which K responded *all the time*. His new-found justification may have given K further ownership of the idea he had supported throughout much of the problem-solving session. K's possible feelings (ownership, smart) could have been a result of satisfying his motivating desire to demonstrate how smart he was while working on this problem with others.

Prior to looking at K's questionnaire responses, our analysis of the episode above suggested to us that the LHSIA and the IRYW structure may have been active for K. His words and actions indicated a motivating desire to display his knowledge, or his smarts, since he had what he believed to be the correct answer. K seemed steadfast in his belief that the ten-block high tower had a total of forty-six blocks, which is the correct answer. Based on the video evidence alone, we acknowledge that other motivating desires or engagement structures may have been at play here. However, the questionnaire results, as shown in Tables 1 and 2, also suggested that the LHSIA and the IRYW structure was active for K at some point during the problem solving session. To help highlight the nuances of these engagement structures, let us turn to an episode of small group work in another classroom on the same problem.

Group II, Class 2

Consider the other group of students, in Ms. S's class: T, A, and C (all female). About 13 minutes into the problem-solving session, the girls were still trying to understand the structures as depicted in the task. In the moments before the episode below, T had been working on the problem aloud, and had determined a total number of blocks for the five-block high tower. She incorrectly stated that there are twenty-five blocks in this tower, instead of only twenty-one. C and A seemed confused, so T started to explain what she was saying and writing. T started to explain how she arrived at her solution for the five-block high tower, but then seemed to realize that C and A did not understand how the five-block high tower was constructed. Again, the analysis is weaved throughout the continuous episode and we focus primarily on T.

[T has drawn on her page a representation of the 5-block high tower. C and A keep their focus on T and her paper throughout the interaction below.]

T: Now it's a 5-block high tower. [referring to her drawing]

C: So count the whole thing? [emphasizes "whole" as she asks if counting is the next step]

T: [counting the blocks, pointing with her pencil as she goes along] One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty, twenty-one, twenty-two, twenty-three, twenty-four, twenty-five. [says "twenty-five" with satisfaction, possibly implying the question, "Do you see that?" to C and A]

T: Five, ten, fifteen, twenty, twenty-five [counting again, pointing to her drawing on her paper]

[A uses her pen to point to T's paper while T is counting]

C: Oh, now I get it. [brief pause] So you're trying to say, that, like, add, like five blocks to each, like, [pauses while pointing to the diagram from the task on T's paper, trying to understand T's explanation] like, this one.

T: Yeah. No. [T first appears to validate what C was saying, and interrupts herself. She continues explaining again.]

T: [pointing to the diagrams on her paper using her pencil] Each, like, say, each set. Say if we call this set one, call this set two, this set three, this set four, and this right here, set five. [drawing circles on blocks on Figure C, perhaps to denote each leg and the height into what she calls sets (see T's work)] Right? [C: (agreeing) Yeah.] Each set has five blocks (holds up five fingers) [C: Oh! (indicates understanding)] There's five blocks for each set. [C: Okay.] So if

there's five blocks for each set, then, add together, and you count it up, there's twenty five blocks in all. [T's tone is patient and controlled. T continuously makes eye contact with C and sometimes A. T settles back into her chair.]

C: All right.

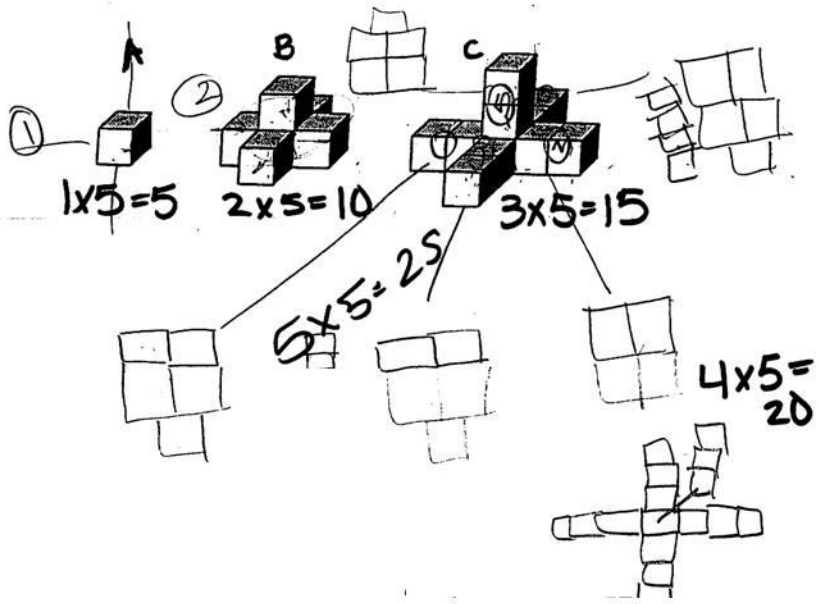


Figure 2: T's drawings

In the dialogue above, T appeared to demonstrate a motivating desire to teach another student something, particularly her two group mates. She began by stating her answer to the total number of blocks in a five-block high tower and included a brief explanation by counting the blocks. Based on the episode above and T's drawings on her paper (see Figure 2), she seemed to believe that the total number of blocks in a five-high tower was twenty-five, which it appears she got by multiplying five times five. However, the answer is twenty-one, which can be determined by multiplying four times four (for the legs on each side) and adding five (the height). T seemed to draw a similar correct representation of a four-block high tower but still claimed there were twenty blocks in the structure, rather than sixteen.

T appeared to realize her two classmates did not exactly understand her solution, so she tried to explain her answer in a different way. Not only did T try to use a different explanation, by using the word "set" to describe the height and each of the legs, but her tone of voice was very patient and controlled. She may have realized that simply showing her classmates how to count up the blocks was not sufficient for them to understand, however, we cannot know that for sure. It seems her actions, particularly her persistence in showing her classmates her solution, were governed by a motivating desire to help her classmates, which is associated with LMTY. T gave a response of *all the time* to several questionnaire items which are associated with LMTY: "I wanted to teach another student something that I knew that the other student did not know," and "I helped someone see how to do the math." By describing the tower structure as several sets of blocks (see the circles dawn on tower C, in Figure 2), T attempted to show another student how to "do the math," or in this case see twenty-five as the total number of blocks in the five-block high tower. Though A did not say much, she was always looking at T while she provided her explanations. C often asked questions or interjected to demonstrate whether she understood

T's solution or explanation. Both C's and A's actions may have led T to respond *all the time* to the questionnaire item, "Others listened carefully to my ideas."

Table 3: Questionnaire items which may indicate Let Me Teach You structure

Questionnaire Items-Statements	***T (f)**
I wanted to teach another student something that I knew that the other student did not know. (motivating desire)	All the time
I helped someone see how to do the math. (implementation action)	All the time
I listened carefully to the ideas of someone I was trying to help. (implementation action)	All the time
I gave helpful suggestions. (implementation action)	Often
Others listened carefully to my ideas. (potential outcome)	All the time
Thoughts (Yes/No)	
I like teaching this person things that I know.	Yes
Some Associated Feelings	
Interested	Somewhat
Respected	Very much
Successful	Very much
Confident	Very much

As the one student in this group of girls who found an answer to one of the task questions, T appears to have developed a sense of ownership of the mathematics. She created a representation on her paper, which she used to help describe her (incorrect) answer of twenty-five for a five-block high tower. She then drew on the given diagrams to help with her explanation. She repeatedly referred to her drawings and markings when explaining her ideas to C and A. She had done some mathematics, and appeared to believe she had an understanding of the problem. When it seemed her two classmates did not share this understanding, T began to describe the solution in different ways, possibly acting in service of LTMY.

In the continuing episode, T realizes that C and A may be having difficulty understanding how the towers are put together, as indicated by the diagrams in the task.

C/A [voices overlapping, the rest is inaudible]: So this is… This is when…

T: No [with hesitation in her voice, perhaps she is not sure how to answer their questions. T leans forward in her chair.]

C: Yeah, this is the one. So you have to add five to there, right? [A: Yeah. (softly)] No, four?

T: Okay. So this is one [referring with her pencil to figure A]. This is the two-block. [referring to figure B and making a note on her paper] You see the bottom one, you're basically making the two-block, right? This is three-block, bottom, dum, dum ["dum" refers to the two blocks on top of the middle bottom block]. That's the three-block. [pointing with her pencil to figure C] If this was the four-block, it should be three squares going up like that. [T keeps her eyes on her paper, as she writes her notes.] That should be a four-block.

There should be four on each side. [T seems to be explaining the construction of the towers.]

C: Oh! [drawn out, implying understanding] Now, I get it.

T: Five blocks.

C: So this one is five-block, right? This one, for number C. [C says with question in her voice, as though she is looking for validation]

T: Well, no. This is one. This is the one-block, right. [T looks at C, and puts up a finger for one.] See, it's the one-block because you see how there's only one block in it, right. One block tall, right? This is a two-block, because the bottom – don't forget about the bottom part. [pointing at figure B, see task] That's a block right there. The bottom part added this one makes this two, and the bottom makes this two, the bottom makes this two, makes this two. [T is pointing at figure B with both her pencil and her finger, indicating that the height and each leg has a length of two if you include the hidden block. She then puts up two fingers.] That means there's two blocks tall, two block high tower. Right? And, this is only a three-block high-tower [puts up three fingers] because the bottom [T and C point to figure C] and the two, make it three high tall [uses her two hands to indicate height].

At this point, T seemed to believe she needed to explain how the towers were constructed, as given in the task. It may be that T saw that if the others did not understand how to construct the towers, they would not be able to correctly count the total number of blocks in specific size towers (i.e., a five-block high tower and a ten-block high tower). C had suggested that figure C, which was a three-block high tower, was the desired five-block high tower. On her questionnaire, T responded *all the time* to the item, "I listened carefully to the ideas of someone I was trying to help" which appears to have been taking place here. By listening to C attempt to articulate her questions and understanding of the problem, T was able to try to continue with her explanations. Though T first attempted to explain how she (incorrectly) arrived at a total of twenty-five blocks, she believed that neither C nor A quite understood how the towers were constructed. In order to count the total number of towers, T seemed to realize that first understanding the procedure to construct the towers was important. Therefore, T modified her explanations, possibly in service of her desire to teach another student something. We infer that a shift occurred from helping her classmates understand her solution to helping her classmates understand the problem. Her actions continue to appear to be in service of the LMTY structure.

In the above portion of the episode, both T and C, in particular, appeared to persevere in a quest for understanding the mathematics problem. C seemed committed to understanding the problem and persisted with her questions. This may have encouraged T to persist in explaining the problem to her classmates. Had C taken a different course of action, such as give up or copy down T's answers, T's motivating desire may have shifted, evoking a different engagement structure. Instead C asked more questions, encouraging T's possible motivating desire to "teach another student something that I knew that the other student did not know," a questionnaire item to which T responded *all the time*. C's questions and statements such as, "I get it," may have contributed to T responding *very much* to questions if she felt "respected" and "successful".

In the entire episode of T, C, and A above, T appeared to make several efforts to explain both her solution to the total number of blocks in the five-block high tower and the set-up of the problem itself. Prior to reviewing T's questionnaire responses, T's behaviors suggested that

LMTY was an active engagement structure for her throughout this episode. T's actions of explaining her solution, her patient tone of voice, repeating those details, and going back to explain an earlier step to justify the construction of the towers all suggest that she was motivated by the desire to teach others some part of the mathematics, and to help them understand the math. When we examined T's questionnaire responses (shown in Table 3), we noticed she responded *all the time* to questionnaire items such as, "I wanted to teach another student something that I knew that the other student did not know," "I helped someone see how to do the math," and responded that she had the thought, "I like teaching this person things that I know" during the problem-solving session. Therefore, we infer from T's questionnaire responses and her actions that she was motivated to help others, she took actions to teach her classmates, and she believed that her two classmates listened to her ideas.

Discussion

The two episodes above depicting two different small groups discussing the details of the problem and its solution are meant to illustrate the three related but distinct engagement structures that we present in this paper. Both K and T appear to believe they have correct solutions and they both share those answers with their group mates. However, our observations combined with the students' questionnaire responses allow us to suggest that K and T were driven by different motivating desires. We suggest that the LMTY engagement structure was active for T. Her classmates seemed to express confusion, and her actions and questionnaire responses indicate that she wished to help them and teach them.

In contrast, it appears to us that the IRYW and the LHSIA engagement structures were active for K. One of his classmates also expressed confusion, but K continued to state his answer rather than explain it at this time during the problem-solving session. K's actions appeared to either help confirm his solutions or provide another with answers he already had. He appeared to attempt to correct another classmate's incorrect suggested answer, rather than explain why one answer was correct and the other was incorrect. His questionnaire responses on items associated with LHSIA also indicated that K was motivated by the desire to show he was smart and that his actions were in service of satisfying that desire. We have mentioned that the questionnaire has been revised since it was administered to these students. Included in the revised questionnaire are additional items to help us infer whether the IRYW structure may be active, as this version had few items, compared to the number of items associated with either LMTY or LHSIA. Yet, with the exception of one item ("I wanted you to admit you were wrong and I was right"), questionnaire data allow us to suggest that IRYW may have been active for K throughout this episode and at other times during this problem solving session.

A primary difference between K and T appears to be the amount of explanation given about the problem or the solution as well as the way in which those explanations were offered. T persisted and continued to give C and A explanations, until they were able to move on to the next step. One of K's classmates, R, already seemed to agree with him (that the ten-block high tower required forty-six blocks instead of the fifty D suggested), and though D expressed confusion, K continued to state the answer, rather than provide further explanation. We acknowledge that the social environment – the classroom, with the teachers, the classmates in the group, and other factors which may be unknown to us – may have also influence both K's and T's motivating desires at these moments. In a different set of circumstances, one or both of these students may have experienced a shift in their motivating desire and subsequently the active engagement

structure. For example, had D been more persistent with his questioning (i.e., Why does a ten-block high tower have forty-six blocks?) or asked different questions, K may have shifted from a perceived desire to be seen as smart toward a desire to help his classmate understand this concept.

Significance

We have presented three of several engagement structures as well as corresponding examples from our classroom data. We look, not only at several episodes via video, but also at the students' questionnaire responses to better understand the interactions between students as they work together on a mathematics problem. Both qualitative and quantitative data have their own limitations. The quantitative data came from the self-report questionnaire the students completed. Any time we rely on self-report data, we run the risk that the participants may respond in a way that may not truly reflect their experiences, perhaps to present themselves in a different light, or to provide answers they believe the researchers would prefer. At the same time, the qualitative analysis of the video and audio data is subject to the interpretation of the researchers. Our hope is that with both the questionnaire data and video evidence we can infer the activation of an engagement structure, though we acknowledge that possibility that questionnaire data or classroom observation alone may not provide sufficient evidence.

In providing classroom episodes, we attempt to illustrate possible manifestations of "Let Me Teach You," "I'm Right, You're Wrong," and "Look How Smart I Am." We recognize that in all three engagement structures, a student may appear to be providing some explanation or help to a classmate. Future work will focus on discerning some differences which may help us understand different ways students may interact when working in small groups. In learning more about these and other engagement structures, we hope to understand a range of peer interactions in a variety of situations. Our goal is to learn how to promote successful peer interactions, which contribute to conceptual understanding of mathematics for all members working in a small group.

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