Measuring Engagement Structures in Middle-Grades Urban Mathematics Classrooms [1]

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Abstract

Engagement structures are idealized psychological constructs that have been proposed as a way of understanding students' affect in classroom situations involving conceptually challenging mathematics. Efforts to infer and measure such structures were based initially on classroom videotapes and retrospective interviews with students. Here we focus on the design of and preliminary results from a questionnaire administered immediately following problem solving activity. Encouraged by our findings to this point, we describe the design for an improved, more comprehensive instrument to assess the activation of various engagement structures in mathematical contexts.

I. Engagement and Mathematics Learning in Schools

> "Certainly the knowledge, beliefs, decisions, and actions of teachers affect what is taught and ultimately learned. But students' expectations, knowledge, interests, and responses also play a crucial role in shaping what is taught and learned. For instruction to be effective, students must have, perceive, and use their opportunities to learn. (Kilpatrick, Swafford, & Findell, 2001, p. 313).

Our research over the past several years has focused on the critical issue of student engagement in middle school mathematics. In particular, some of us have been studying the affective interactions of students working in groups on conceptually challenging mathematics problems in urban, middle school mathematics classrooms. This work has been situated in low-income, predominantly minority communities in the state of New Jersey. As a result, we developed some theoretical ideas around what we now call "engagement structures" (Alston et al., 2007; Epstein

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et al., 2007; Goldin, Epstein, & Schorr, 2007; Schorr et al., 2010-a,b; Schorr, Warner, & Arias, 2008; Schorr, Epstein, Goldin, Warner, & Arias, submitted for publication).

In the present article, we first describe this theoretical construct briefly. We then present some preliminary findings using a research instrument developed for the purpose of inferring or confirming the activation of engagement structures while students worked in small groups. This phase of the study generally supports the validity of the construct. Finally, we describe the design for an improved, more comprehensive instrument to assess the activation of a wider set of engagement structures in mathematical contexts.

Historically, measurement of mathematical affect has been approached in several distinct ways. First discussed was mathematics anxiety (based on a psychological perspective), with several researchers creating instruments or subscales to other instruments. The Mathematics Anxiety Rating Scale (Richardson & Suinn, 1972) continues to be used today (Capraro, Capraro, & Henson, 2001). A second focus has been attitude (based on a sociological perspective); an important instrument has been the Mathematics Attitude Scale (Fennema & Sherman, 1976). This instrument included an anxiety factor, but added several other factors. The justification for focusing on anxiety was an underlying assumption that high anxiety was bad or detrimental to learning, and low anxiety was good or beneficial. More generally, in the domain of attitude, positive feelings have been considered good, and negative feelings bad. However, research seems to show that a degree of mathematics anxiety is important to mathematics learning; and that not all positive attitudes are related to learning success, while not all negative attitudes are related to failure. What is beginning its rise in prominence is the study of more complex affective structures, including structured beliefs and meta-affect, and correspondingly more complex relationships (e.g. Maass & Schlőglmann, 2009).

An increasing body of research in mathematics education investigates relationships among affect, motivation, engagement, and beliefs. Some of this work highlights the importance of these variables at the middle school level. Affect, as we refer to in this paper, includes patterns and structures of emotions, attitudes, beliefs, and values (DeBellis & Goldin, 2006; Goldin, 2000, 2002; McLeod, 1992, 1994; Zan, et al., 2006; see also Dai & Sternberg, 2004, Leder, Pehkonen, & Tőrner, 2002; Lesh, Kaput, & Hamilton, 2007; Maass & Schlőglmann, 2009, Schutz & Pekrun, 2007, and references therein). By motivation, we mean an individual's desire, interest, sense of goal or purpose, inspiration, or aspiration to engage in or persist in an activity (see also Alderman, 2008; Eccles, Wigfield, & Schiefele, 1998; see also Schunk & Zimmerman, 2008, and references therein). We shall use the term engagement to refer to an individual's devotion of attention and purpose to a specific activity on a particular occasion. On different occasions, students' affect and motivation may foster or inhibit their mathematical engagement, concentration, persistence, and eventual success.

Let us consider engagement in greater detail. In general, engaged people are motivated by four fundamental goals that satisfy specific human needs – success, curiosity, originality, and relationships with others; these are related to needs for mastery, understanding, self-expression, and involvement with others respectively (Strong, Silver, & Robinson, 1995). Schlecty (1994) find that students who are engaged display three characteristics: (1) attraction to their work, (2) persistence with their work regardless of difficulties and barriers, and (3) pleasure in completing their work. Research shows that students who are engaged in class, spend time on their homework, and steer clear of disruptive conduct in schools demonstrate larger gains in learning (Ainsworth-Darnell & Downey, 1998; Farkas, 1996) and greater achievement (Lleras, 2008a; Rosenbaum, 1980).

"Disengaged students disrupt classes, skip them, or fail to complete assignments. In contrast, engaged students make a psychological investment in learning and try hard to learn what a school offers" (Newmann, 1992, p. 2-3). A positive relationship exists between student engagement and academic achievement (Finn, 1993; Greenwood, 1991; Marks, 2000). Park (2005) finds positive effects for student engagement consistent regardless of SES or minority status on student growth in mathematics; however, classroom level factors such as class size, teachers' degree, certification, experience, authentic instruction, and content coverage are not significant predictors of student mathematics achievement growth. These findings suggest that student engagement should be emphasized in school settings.

Data from a National Educational Longitudinal Study by Lleras, (2008b), involving over 7000 students, indicate that student engagement along with other factors positively affect students' learning over time, with effects taking place within both high- and low-minority schools. She concludes by noting that "less desirable habits and lower skills in middle school translate into even greater gaps in skills, habits, and achievement in high school" (p. 906). Thus student engagement in middle school should be emphasized as an important factor in schools, and a factor that should influence educational policy.

A socially-enhanced middle school learning environment is one that is arranged in a way that enhances learning for those who behave in a way that is well-suited within a school setting. This environment may focus more attention on students who display a greater eagerness to learn or more willingness to obey the rules of the classroom. Additionally, it could be displayed as "the use of group-learning environments where a willingness to participate actively or otherwise show engagement in the group produces enhanced learning" (DiPrete & Jennings, 2009, p. 27).

Thus, the authors of the Final Report of the National Mathematics Advisory Panel (2008) note the importance of "…greater engagement in mathematics learning and, through this engagement, improved mathematics grades and achievement" (p. 31).

II. Engagement Structures

In the first phase of research leading up to the present paper, the Rutgers University group conducted an exploratory empirical study in three urban classrooms in predominantly minority, low-income communities. We videotaped entire classes as well as small groups of 7th- and 8th-grade students working together on a variety of conceptually challenging mathematics problems. Different cameras captured different groups of children and different views of the classroom and the teacher. We also made use of stimulated-recall, retrospective interviews with selected children (Alston et al., 2007; Epstein et al., 2007). We analyzed much of the data making use of four "lenses": the flow of mathematical ideas, key affective events, students' social interactions, and teacher interventions. We concluded that a new construct was needed to account for and understand the complex, dynamical interactions that seemed to recur. That construct was initially termed an "archetypal, affective structure" – an idealized, recurring, highly affective dynamical pattern, inferred from observing classroom and interview tapes and validated through questionnaire responses.

There is a rather close analogy between terms such as "affective structure" or (more specifically) "engagement structure" as we use them, and the more familiar term, "cognitive structure." Affective structures, or engagement structures, refer to stable configurations within individuals that involve emotional feelings, where one or more affective pathways (i.e., characteristic sequences of emotional feelings) interact with each other and with cognition. The adjective "archetypal" is intended to suggest the idealized nature of our descriptions, together with the universality or near-universality of presence in individuals of the structures we describe.

Engagement structures are affective structures whose features are oriented toward a particular motivating desire. They involve recurring patterns that form a kind of behavioral/affective/social constellation, situated within the individual but becoming active in characteristic social situations. A set of parallel strands, woven together, constitute the fabric of such a structure. We regard these as simultaneously-occurring, mutually-interacting components (Goldin, Epstein, & Schorr, 2007):

(1) a characteristic goal or *motivating desire*, evoked by particular circumstances in the social environment,

(2) a characteristic *pattern of behavior*, beginning in response to the circumstances evoking the motivating desire, and culminating (possibly) in a characteristic outcome,

(3) a characteristic sequence of emotional feelings, or affective pathway,

(4) expressions from which affect may be inferred, emotionally expressive words, eye contact and facial expressions, posture and "body language," movements involving others, interjections and exclamations, tears and laughter, blushing, etc.,

(5) information or meanings encoded by the emotional feelings,

(6) meta-affect, which (in analogy with metacognition) may include feelings about feelings, feelings about cognition about feelings, and self-monitoring of affect,

(7) characteristic self-talk or inner speech, responding to and evoking emotional feelings,

(8) interactions with the individual's systems of beliefs and values,

(9) interactions with the individual's structures of self-identity, integrity, intimacy, or other affective structures,

(10) characteristic problem-solving or decision-making strategies and heuristics.

Let us describe several examples of such engagement structures that were inferred from early analyses of classroom videotapes and retrospective interviews, and that entered into the creation of a questionnaire instrument designed to measure their activation.

(a) "Get The Job Done" (GTJD): The student's motivating desire is to fulfill a sense of obligation to complete an assigned mathematical task, to correctly follow instructions given, or to meet a commitment. A sense of satisfaction follows from having fulfilled the commitment, if this occurs (not necessarily from having achieved a mathematical understanding). In a group context, the student may seek to enlist others in accomplishing the task.

(b) "Look How Smart I Am" (LHSIA): The motivating desire is to impress others (or, possibly, himself or herself) with the student's mathematical ability, knowledge, intelligence, or genius. Satisfaction follows from obtaining recognition of how impressive the student's thinking or achievement is, if this occurs.

(c) "Let Me Teach You" (LMTY): Here a student's awareness, insight, or relevant mathematical knowledge leads to the motivating desire of sharing this with one or more other students. Satisfaction results from the other student(s) learning and/or appreciating the help.

(d) "Don't Disrespect Me" (DDM): The motivating desire here is to meet a perceived challenge or threat to the student's dignity, status, or sense of self-respect and well-being. In the context of a challenge to the student's mathematical idea, the need to "save face" may override the understanding of mathematical concepts.

(E) “Check this Out” (CTO): The motivating desire comes from the student’s realization that solving the mathematical problem can have a “payoff” that the student wants. The resulting attention to the task can then heighten (intrinsic) interest in the task itself, or (extrinsic) interest in a (non-mathematical) payoff.

(f) “I’m really into this” (IRIS): Here the student becomes motivated by intrinsic interest in the very activity of addressing the task, leading to the experience of “flow” (Csikszentmihalyi, 1992) where the student “tunes out” other elements of the environment. Satisfaction may be derived from achieving deep understanding, solving a difficult problem, or from the experience of fascination during profound engagement.

(g) “Stay Out Of Trouble” (SOOT): The student’s motivating desire here is to avoid interactions that may lead to conflict with peers or the teacher, or emotional distress such as embarrassment, humiliation, or anger. Aversion to risk supersedes involvement with the task’s mathematical content.

(h) “Pseudo-Engagement” (PE): Here the student desires to look good (to the teacher or to peers) by appearing to be engaged with the mathematical task at hand, although the engagement is not actual.

It is important for us to distinguish our concept of engagement structures from the different but important notion of motivational orientations. In doing so, we want to highlight and sharpen in the present context the distinction between relatively long-term, stable characteristics of the individual, or “traits,” and the much more rapidly changing, immediately active characteristics that define a person’s present “state.” Much literature considers features of an individual’s motivation, such as his or her achievement goal orientations, to be in the longer-term (or slowly changing) category (e.g., Schutz & Pekrun, 2007; Schunk & Zimmerman, 2008; and references therein). These include “mastery-approach goals,” focusing on achieving (mathematical) understanding of the material, as opposed to “performance-approach goals,” focusing on achieving as much as or more than others. Linnenbrink (2007) proposes “... a triarchic model of reciprocal causations ... in which there are reciprocal relations among achievement goal orientations and affect, affect and engagement, and achievement goal orientations and engagement” (p. 122). Another important distinction in the literature on motivation is between “intrinsic” and “extrinsic” rewards, the former being associated with the “informing function” of reward and the latter with the “controlling function” (Zimmerman & Schunk, 2008).

Of course, one can immediately identify certain engagement structures as more likely to be activated in association with mastery-approach goals and/or intrinsic motivation (e.g., IRIT) and others more likely to be activated in association with performance-approach goals and/or extrinsic motivation (e.g., LHSIA). But (in analogy with cognitive structures) we hypothesize most or all of the engagement structures we describe to be present in most or all individuals. Different engagement structures can become active (in the same individual) under different social conditions. Achieving optimal mathematical engagement is then a matter of creating a classroom environment in which engagement structures are activated that are appropriate to the learning occasion.

III. Methodology and Prototype Questionnaire Development

As described above, the engagement structure concept was developed during the analysis of qualitative data from the first, exploratory phase of a study of affect in urban middle school classrooms. Following that phase we devised a prototype questionnaire for use in the next phase of the study. Here our main goal was to examine qualitatively the instances of alignment or non-alignment of questionnaire responses with evidence from classroom videotapes.

In this phase of the study, all students in different classes worked on the same task – the “Building Blocks” task. We deemed that the task, while algebraically elementary, would be conceptually complex for most of the students in our population (using the criteria cited in Stein, Smith, Henningsen, & Silver, 2000). This problem was adapted (with modification) from one called the “Building Blocks Dilemma” that had originally been selected by one of the teachers during the first phase of research leading up to the present paper.[‡‡]

The teacher distributes a problem sheet that includes the following printed information:

> “I was constructing towers as you see below. I noticed that each time I made the tower higher, I added more blocks on the sides. I would like to know how many cubes I will need to build a 5 block high tower, a 10 block high tower, and a 100 block high tower. Generalize if you can on how many blocks I will need for any size tower.”

The sheet also contains three figures depicting a one block high tower, a two block high tower, and a three block high tower, as shown in Figure 1.

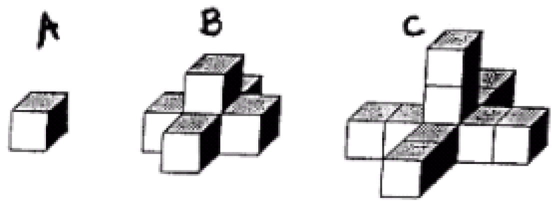

Figure 1. Illustration for the “Building Blocks” task

The students worked on this problem in groups of three. At the conclusion of their work, they completed the prototype questionnaire.

We developed, piloted, revised, and administered a questionnaire designed to help us infer whether specific engagement structures were active or not (for individual students) during the class. This questionnaire, provisionally named the “Rutgers University Mathematical Engagement Structures Inventory” (RUMESI) is the one about which we report in this paper. It includes 42 items describing *thoughts or experiences a student might have* during the class, with responses requested according to a 3-point Likert scale: 0 (never), 1 (some of the time), or 2 (all of the time). These are the items listed in Table 1 below. In addition, 22 items comprise a list of words describing *emotional feelings* a student might have during the class, 11 positive feelings and 11 negative; again with responses according to a 3-point Likert scale: 0 (not at all), 1 (somewhat), or 2 (very much).

We videotaped and audiotaped the students, this time in each of the small groups. At this stage in the research, our primary goal was to ascertain whether we could find correspondences

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[‡‡] Exemplars K-12 (2004), http://www.exemplars.com/resources/alignments/impact_course01.html

between events we viewed in the videos and questionnaire responses. We also wanted to explore whether the hypothesized engagement structures that we inferred from students' observed behavior could also be detected through statistical analyses of questionnaire data. Schorr et al. (submitted for publication) incorporates a qualitative report of some of our findings.

Here we report on normative, reliability, and validity data for the prototype questionnaire.

IV. Analyses of the Prototype Questionnaire

We used a two studies format to examine the psychometric properties of the instrument. The first sample was used to examine the structure in order to achieve a parsimonious model. Because the instrument had been designed around a theoretical framework that included five engagement structures in the initial study (GTJD, DDM, CTO, IRIT, and LHSIA), we used exploratory factor analysis to reduce the number of items and/or refine some items. In the second study, we use confirmatory factor analysis to examine the refined model. In each study, distinct and disjoint sets of participants were used.

Two middle grades samples (N=450 and N=425) participated in assessing the psychometric properties of this instrument. In Study 1 the structures were defined and items were written for each structure, with input from and in consultation with psychologists, mathematics education researchers, mathematicians, middle grades teachers and middle grades students. After external validity concerns had been addressed through collaboration with these major constituents, the items were pilot tested, following a protocol developed to ensure uniformity of administration and consistency within the data [see Note 1].

The sample was purposively selected. In Study 1, inner city students in a major urban center in Northeastern United States were used, approximately 52% female, mostly African American and Latino. The students' teachers had been participating in a large-scale professional development funded project. In Study 2, the refined instrument was used with a new purposively selected sample of inner city students in a major urban center in Southwestern United States, whose teachers were participating in a large scale professional development funded project (TEA0969441071100003). This sample was 54% female, mostly African American (58%), with 32% Latino, 9% White, and 1% other.

A. Study 1: Item Reduction and Parsimony

For the exploratory factor analysis (EFA), all 42 items were used in three reasonable models. In the first model, we used two strategies for estimating the number of factors extracted: the eigenvalue greater than 1 rule, and scree plot test (Henson, Capraro, & Capraro, 2004). The second model was set to extract the seven factors of Let Me Teach You (LMTY), Get The Job Done (GTJD), Don't Disrespect Me (DDM), Check This Out (CTO) and I'm Really Into This (IRIT), Look How Smart I Am (LHSIA), Stay Out of Trouble (SOOT), and Pseudo Engagement (PE). For the third model we hypothesized a combined structure with only five factors, not only combining CTO and IRIT into a single factor, but distributing SOOT and PE across two of the other factors.

The data for these EFA analyses were comprised of 443 complete cases; but 21 cases were also eliminated due to patterns interpreted as invalidating the data responses. Three general patterns were observed: marking the same response for all items, creating patterns of responses 1, 2, 3, 4 . . . or 1, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4 . . ., and finally all one response.

For all EFA analyses, principal axis factoring (PAF) or common factor analysis were used (Thompson & Daniel, 1996). The fundamental reason for choosing PAF was that it uses estimates of reliability on the diagonal of the variables because this addresses the assumption that error variance should not be considered as part of the analysis. Communality (*h*2) coefficients, or the proportion of variance in the variable that is reproducible by the factors, are often used on the diagonal as lower bound estimates of reliability. That is, PAF attempts to remove error from analysis; however, this is not a concern when reliability estimates are high as they will approach 1. Because the factors are correlated we chose an oblique rotation, which necessitated the interpretation of both the factor pattern and factor structure matrices.

EFA Model 1

In Tables 1 and 2, the factor structure and factor pattern coefficients show how items were allocated to each factor. In this model 12 factors were extracted. The first eight accounted for 49% of the variance, far below the cut value. The scree plot and eigenvalue greater than 1 rule indicated a seven-factor solution. However, hypothesized items, for the most part, loaded together. The Cronbach's alpha reliability for the structures as predicted were: .78 (LMTY), .65 (GTJD), .58 (DDM), .88 (CTO), .83 (IRIT), .79 (LHSIA), .63 (SOOT), and .51 (PE). Using case diagnostics, items 1, 2, and 7 were deleted. However, these items were representative of IRIT and CTO, two of the structures with the largest reliability coefficient. We examined the distribution and found 16 cases, which when deleted brought the distribution within the skewness and kurtosis parameters for a normal distribution.

Table 1

Factor Structure Matrix for RUMESI (all items)

Variables	Component							
	1	2	3	4	5	6	7	8
Q40: I enjoyed learning math	.691							
Q3: As I made progress I became more interested in understanding the math CTO	.684							
Q2: I was fascinated by the math today IRIT	.666							
Q4: I was so into my work that I tuned out what was going on around me IRIT	.613							
Q5: I felt that learning the math today would benefit me or pay off for me. CTO	.520							
Q10: While working on this problem I thought of myself as a mathematical problem solver	.505							
Q41: I achieved a good understanding of the math we worked on today IRIT	.456							
Q30: I wanted to make sure that all the required work was completed GTJD		.705						
Q32: I worked on getting the answer to the problem		.618						

GTJD							
Q31:The most important thing for me was getting the answer to the problem GTJD		.600					
Q35: I felt relieved when all the work was done GTJD		.501					
Q34: I wanted the teacher to think I am a good student GTJD Conclusion		.477		.358			
Q7: I realized that if I worked hard on the problem I could figure it out CTO	.355	.454					
Q36: I felt proud about what I accomplished CTO	.356	.451					
Q17: I listened carefully to the ideas of someone I was trying to help. LMTY			.653				
Q19: Others listened carefully to my ideas LMTY			.597				
Q18: I helped someone see how to do the math LMTY			.585				
Q33: I tried to get members of my group to work to get the answer to the problem GTJD			.531				
Q16: I wanted to teach another student something that I knew that this other student did not know LMTY	.375		.456				
Q11: I wanted people to think that I am smart LHSIA				.681			
Q12: I tried to impress people with my ideas about the problem LHSIA				.641			
Q14: People saw how good I am at the math we did today LHSIA				.578			
Q15: I felt smart LHSIA		.357		.546			
Q13: People seemed impressed with the ideas I shared about the problem LHSIA			.351	.499			
Q38: I was worried that I might get into trouble with the teacher PE					.747		
Q26: I was worried I might do something that would get me into trouble with one or more students. SOOT					.652		
Q24: Some person or group of people tried to disrespect me DDM					.570	.411	
Q28: I hoped people would not pay attention to me SOOT					.559		
Q37: I wanted to look like I was doing work even when I wasn't PE					.456		
Q29: I cared more about feeling OK than about solving the math problem SOOT					.395		
Q21: I argued strongly in support of my ideas Day 1 DDM Continuation 2						.693	
Q20: I wanted to show someone that my way was better LHSIA						.668	
Q22: I had an unpleasant disagreement DDM						.627	
Q23: My ideas were challenged by others DDM						.604	
Q25: I was not going to let someone disrespect me and get away with it DDM Continuation 1						.438	
Q42: I was a lot better at math than others today LHSIA							
Q8: I got stuck trying to solve a math problem today.							.802
Q6: As I worked on the problem I found it challenging							.796

Q9: I felt it was OK to get frustrated and let the teacher see that I felt frustrated.			.556
Q27: I paid attention to the way others were reacting to me SOOT		.358	.530
Q39: I paid attention to things happening in class that had nothing to do with math.			.422
Q1: I concentrated deeply on today's math problem IRIT	.775		

Note: Principal Factor Analysis with a Promax with Kaiser Normalization. Coefficients with absolute value greater than .33 are shown.

Table 2

Factor Pattern Matrix for RUMESI Revised

Variable	Component Structure	I	II	III	IV	V	VI	VII	VIII
Q1	IRIT	.870	.007	.189	-.224	.398	.370	.021	.062
Q2	IRIT	.875	.045	.152	-.164	.062	.400	.410	.128
Q3	CTO	.557	-.018	-.009	-.127	.020	.167	.115	.060
Q4	IRIT	-.655	-.067	.139	-.035	.029	.009	.115	.084
Q5	CTO	.375	.201	-.038	.014	.126	.042	.096	-.012
Q6	nps	.405	.526	.146	.174	.216	.291	.227	.223
Q7	CTO	.660	.294	.035	.343	.360	.237	.166	.075
Q8	nps	.153	.125	.756	.046	.221	.185	.287	.171
Q9	nps	.363	.035	.412	.157	.132	.026	.177	.324
Q10	nps	.795	-.054	.146	-.208	.260	.349	.494	-.087
Q11	LHSIA	.002	.520	.158	.280	.068	.084	.086	-.070
Q12	LHSIA	.170	.912	.163	.090	.142	.298	.125	-.158
Q13	LHSIA	.170	.910	.121	-.065	.028	.025	.063	.042
Q14	LHSIA	-.402	.412	.313	.092	.196	.239	.026	-.038
Q15	LHSIA	-.045	.538	-.138	-.184	.068	.072	.113	-.039
Q16	LMTY	.248	.088	.801	-.062	.019	.148	.133	.056
Q17	LMTY	-.015	.148	--.330	.038	.010	.095	.011	-.162
Q18	LMTY	-.009	.037	-.391	.019	.027	.254	.016	-.030
Q19	LMTY	.010	-.342	.675	.001	.096	.198	.337	-.035
Q20	LHSIA	.231	-.787	.112	.004	.073	.296	.322	.291
Q21	DDM	-.054	-.054	-.012	.783	.030	.049	.088	.165
Q22	DDM	.138	.155	.280	-.471	.045	.051	.133	.154
Q23	DDM	.143	.228	-.043	.677	.132	.003	.112	.069
Q24	DDM	-.001	.290	.023	.925	.165	.197	.158	.005
Q25	LHSI	.145	.613	-.122	.012	.229	.171	.003	.213
Q26	SOOT	.035	.043	.120	.203	.678	.207	.090	.730
Q27	SOOT	.572	.483	.181	- .069	.297	.756	.213	.705
Q28	SOOT	.102	.205	-.028	.014	.550	.065	.220	.456

Q29	SOOT	.248	.199	.030	.187	.883	.157	.027	.668
Q30	GTJD	.006	.083	.105	.223	.229	.963	.307	.521
Q31	GTJD	.784	.397	.456	.243	.745	.224	.298	.428
Q32	GTJD	.043	.042	.668	.623	.859	.549	.829	.162
Q33	GTJD	.244	.045	.021	.340	.454	.870	.012	.268
Q34	GTJD	.415	.099	.128	.077	.034	.740	.595	.034
Q35	GTJD	.065	.282	.162	.268	.251	.444	.162	.151
Q36	CTO	.953	.419	.268	.002	.546	.219	.757	.899
Q37	PE	.092	.244	.164	.146	.040	.139	.955	.398
Q38	PE	.016	-.162	.097	.082	.194	.061	.581	.087
Q39	nps	.335	.043	.808	.115	.024	.175	.154	.379
Q40	nps	.372	.183	.258	.179	.463	.972	.847	.227
Q41	IRIT	.602	.005	.352	.017	.164	.175	.087	.363
Q42	LHSIA	.001	.499	-.025	.118	.103	.061	.080	.315
Trace		.712	1.597	.003	.186	.886	-.006	-.126	.018
% of Variance		28.5	19.2	10.9	9.7	8.2	5.1	3.1	2.3

Note: nps = no particular structure,

EFA Model 2

The number of components was set to seven because of the results from the first analysis. The variance accounted for was 51% for this model. However, the reliability coefficients for some of the structures changed in important ways. For the seven structures they were .69 (LMTY), .63 (GTJD), .74 (DDM), .89 (CTO and IRIT), .81 (LHSIA), .55 (SOOT), and .50 (PE); but the PE and SOOT items included as suggested in the model resulted in a coefficient of .776 for the combined CTO and IRIT structure. We believed this model provided a better fit to the data, but the changes in the item loadings negatively impacted the reliability of some structures. Therefore, item diagnostics resulted in more items deleted to improve the reliability of the structures. We also decided that both SOOT and PE were not sufficiently defined nor saturated with items so the remaining items loading on those two structures would not remain in the final analysis, and those items were also deleted. Below contained in Tables 3 and 4 are the Pattern and Structure Matrices for EFA Model 2.

Table 3
Pattern Matrix for EFA Model 2

Pattern Matrix[a]

Item	Structure	Factor 1	2	3	4	5	6	7
q4	CTO	.706		-.109		-.171	-.128	-.281
q5	CTO	.658	.114			-.117	-.114	-.145
q25	CTO	.616					-.139	-.291
q2	CTO	.581		-.113		.103		
q1	CTO	.515	.264		-.223	.126		
q3	CTO	.379				.163		
q22	GTJD		.875		-.104			
q20	GTJD		.733					
q24	GTJD		.595		-.112			
q23	GTJD		.439		.370	-.156	-.164	.244
q21	GTJD	.201	.351		.227	-.208	.116	.217
q27	DDM				.894			
q26	DDM				.892			
q17	DDM				.685		-.138	.331
q15	DDM				.535		.121	.298
q14	DDM	-.168	.211		.332	.171	.183	.311
q18	DDM				.735			-.314
q16	DDM	-.110	.221	-.115	.570	.119		.234
q19	DDM	-.150	-.165	.103	.630	-.135	.305	.215
q8	LHSIA					.724		
q9	LHSIA	.128				.638		
q10	LHSIA		.109		.101	.433		
q6	LHSIA		.127	.130		.370		
q7	LHSIA	.203			.299	.296	.107	.111
q12	LMTY		-.115	.531	.173			
q11	LMTY	.254	-.163	.313	-.116	.109	.150	.290
q13	LMTY		-.121	.310	.135	.212		

Extraction Method: Principal Axis Factoring. Rotation Method: Promax with Kaiser Normalization.

Table 4
Structure Matrix for EFA Model 2

Structure Matrix

	Factor						
	1	2	3	4	5	6	7
q25	**.688**	.330	.196	.270	.322	-.198	-.198
q5	**.681**	.379	.265	.256	.158	-.203	-.203
q4	**.610**	.158		.223	.123	-.219	-.219
q2	**.607**	.262		.284	.347		
q12	.374	.154	.103	**.585**	.346		
q1	**.572**	.398	.162	.144	.276	-.102	-.102
q10	.523	.299	.238	.219	.270		
q6	.506	.344	.280	.336	.271		
q3	**.456**	.229		.227	.318		
q19	.211		**.438**			.198	.198
q13	.400		**.512**	.323	.394		
q22	.224	**.795**	.139	.186			
q20	.345	**.746**	.185	.304	.192		
q23	.410	**.624**	.291	.485		-.136	-.136
q24	.256	**.595**	.245	.163	.108		
q21	.328	**.473**	.153	.365			
q27	.214	.228	**.888**	.119			
q26	.230	.247	**.886**	.155			
q17	.252	.160	**.629**		.290		
q15	.181	.199	**.554**	.116	.257	.236	.236
q14	.123	.291	**.451**	.117	.304	.288	.288
q11	.325			**.424**	.355	.200	.200
q8	.305			.374	.733		
q9	.417	.167		.344	.692		
q7	.437	.184	.136	.526	.526	.194	.194
q18	-.102		**.728**				.728
q16		.163	**.582**	.213			.582

EFA Model 3

In Model 3, the number of components was set to five because we believed the data could reasonably support five structures with suitable reliability coefficients. We expected that some items would not load satisfactorily or load on the first component but given the deletion of cases we expected a better-fit model. This model accounted for 84% of the variance, and provided a model that argued for the combination of CTO and IRIT. The Cronbach's reliability coefficient for the combined structure was .901, however, when we included the items that moved to CTO and IRIT the coefficient was .776 so we did not retain those items. This model was the best one for the Confirmatory Factory Analysis (CFA), which was the subject of Study 2. Below contained in Tables 5 and 6 are the Pattern and Structure Matrices for EFA Model 3.

Table 5
Pattern Matrix for EFA Model 3

Pattern Matrix[a]

	Factor				
	1	2	3	4	5
q4	.634				
q5	.627				
q19	.619				
q25	.614				
q2	.576				
q1	.543				
q12	.483				
q10	.432				
q3	.390				
q6	.348				
q22		.855			
q20		.714			
q24		.557			
q23		.535			
q21		.433			
q8			.646		
q17			.564		
q7			.550		
q9			.519		

q15			.485		
q14			.445		
q11			.405		
q13			.336		
q27				.905	
q26				.882	
q18					.730
q16					.561

Table 6
Structure Matrix for EFA Model 3

Structure Matrix

	Factor				
	1	2	3	4	5
q25	.693		.355		
q5	.675	.376			
q2	.608		.379		
q4	.596				
q1	.579	.373			
q12	.561		.444		
q10	.522				
q6	.493	.348	.368		
q3	.458				
q19	.447				
q22		.784			
q20		.738			
q23	.371	.636	.333		
q24		.580			
q21		.490			
q7	.413		.636		
q8			.606		
q9	.417		.579		
q17			.521		
q15			.482		

q11			.473		
q14			.455		
q13	.392		.434		
q27				.896	
q26				.877	
q18					.735
q16					.584

B. Study 2: Confirmatory Factor Analyses

To examine the robustness of the final model from the EFA, a separate set of data from 425 middle grades students was analyzed. Four models for CFA were used. The first model was the one best describing the current state of thinking related to mathematical engagement, and was the resultant model from the EFA with the exogenous variables uncorrelated (see Figure 2). The second competing model correlated the exogenous variables (see Figure 3).

The third model incorporated changes suggested in the modification indices that were not in conflict with the theoretical framework (see Figure 4, following page), and the final model hypothesized a super-ordinate structure or the possibility of other structures existent in the data (see Figure 5, following page).

The fit indices are shown in Table 7. Model 3 is the best fit model. Models 1 and 2 have marginal fit indices, and Model 4 was not modeled. Therefore, hypothesized structures of mathematical engagement were correlated and the new data supported the model suggested by Model 3 of the EFA. This replication provides a solid foundation for building on the theoretical model, and generating new items that might better align with the full theoretical framework that separated CTO and IRIT, and that can better distinguish between SOOT and PE.

Table 7

Fit Indices for the Four SEM Models

Model	1	2	3	4
AGFI	.71	.78	.88	--
AIC	1374.40	1075.58	1018.99	810.00
Chi Square	1266.4	947.58	876.99	--
GFI	.76	.81	.91	--
RMR	.08	.05	.05	--
RMSEA	.092	.076	.023	.143

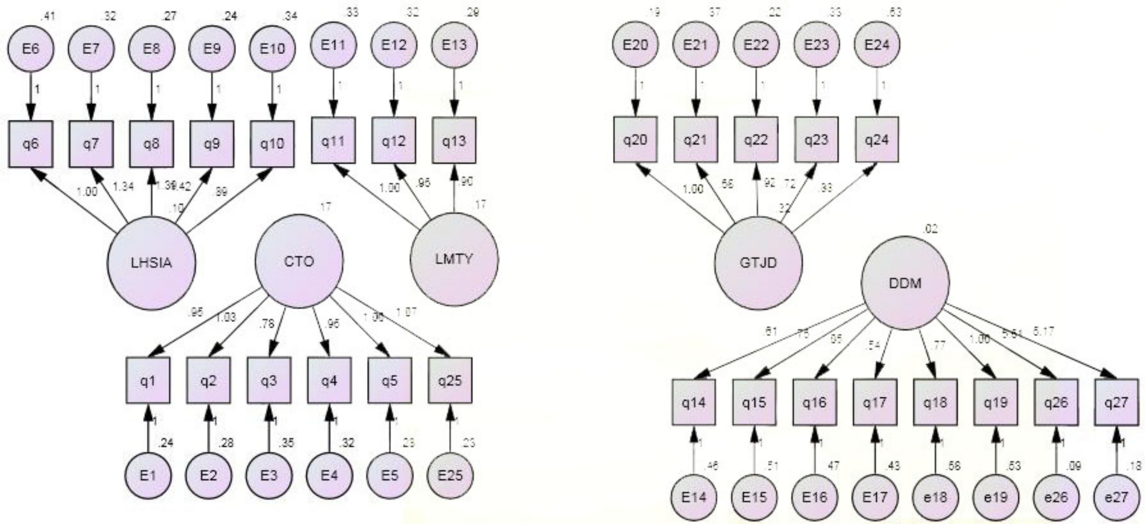

Figure 2. Model 1: Uncorrelated exogenous variables with items.

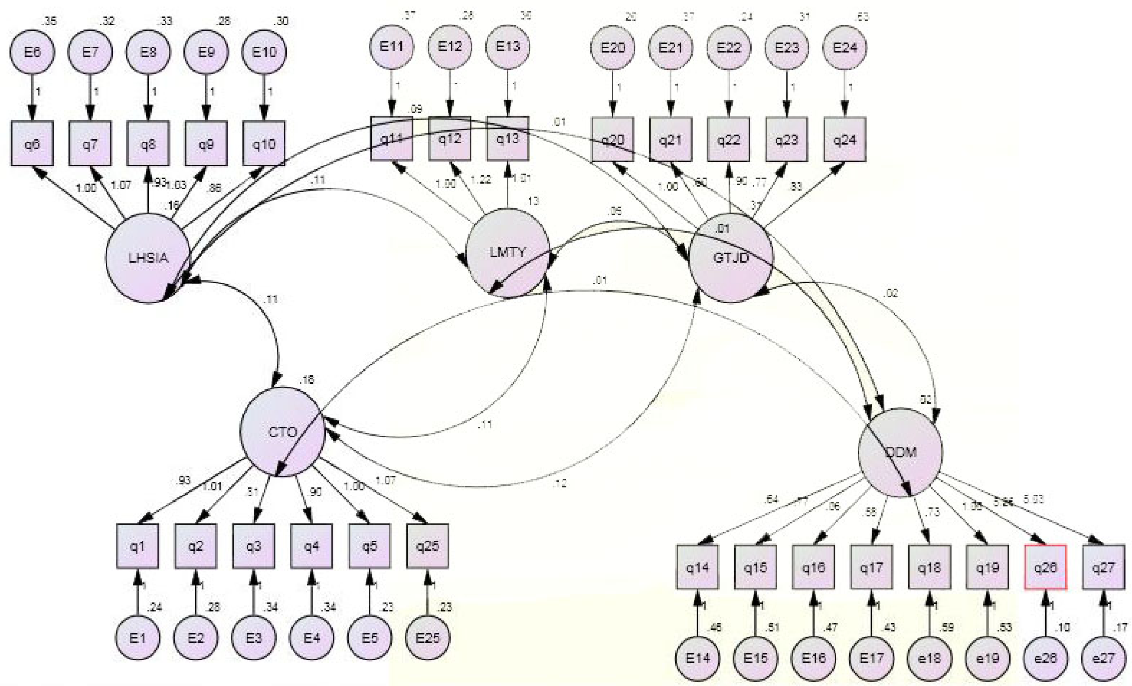

Figure 3. Model 2: Correlated exogenous variables.

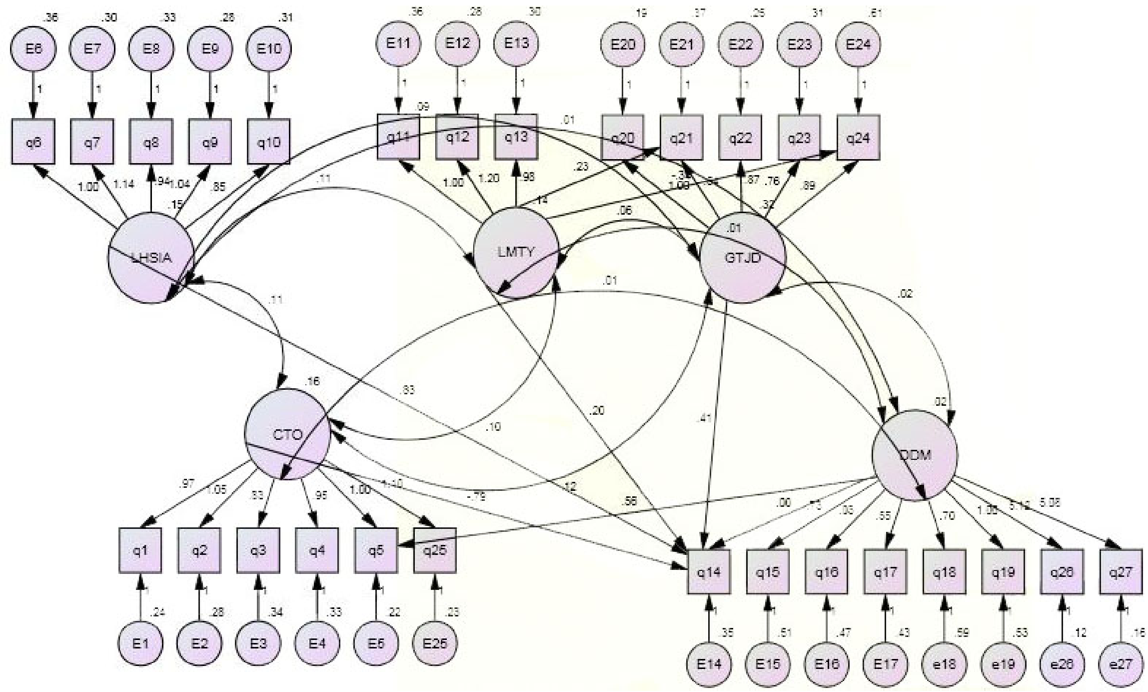

Figure 4. Model 3: Correlated exogenous variables and added paths based on modification indices.

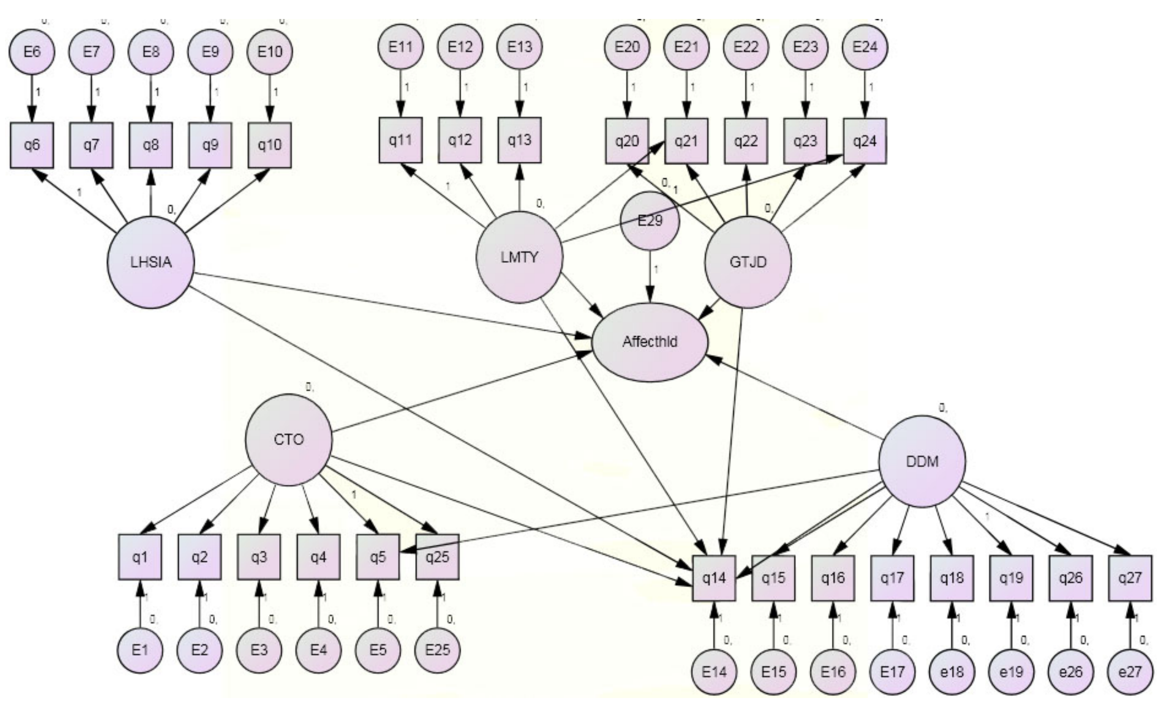

Figure 5. Model 4: Correlated exogenous variables and added paths based on modification indices.

V. Revised Conceptualization and Analysis of Engagement Structures for Measurement

The confirmatory factor analyses based on a prototype of the RUMESI provided sufficient support for the structures we had conceptualized to encourage further development of the instrument. The next phase of our work consisted of developing a more robust conceptualization of the engagement structures and ways to determine through questionnaire items whether a given structure was active for a student during a class session.

We focus now on the following features of engagement structures: (1) the initial *motivating desire*; (2) *action taken* toward fulfillment of the motivating desire; and (3) the *outcome* the student experienced during the class session. To assess whether a given structure was active for a particular student during a given group problem solving session, we seek information about the following: (1) how *much of the time* during the class session the motivating desire was experienced; (2) how *important* the motivating desire was for the student; (3) how much of the time during the class session the student engaged in action whose goal was to fulfill the motivating desire; (4) how important the action toward fulfilling the motivating desire was for the student; and (5) the perceived outcome during the class session.

To this point, we have conceptualized 14 possible structures of interest in connection with students in small groups engaged in conceptually challenging mathematical classroom activity. The *motivating desire* for each structure is associated with a *need* described by Henry Murray (2008) in his book, 70th Anniversary Edition of Explorations In Personality (see Table 4 below for this correspondence). Murray's landmark book, originally published more than 70 years ago, explored the concept of personality structure focusing on a set of manifest and latent needs residing within the individual. Murray's idea was that to understand behavior, it was necessary to comprehend the needs, as well as the *environmental press*, a term he used to describe the situational constraints facing the individual. Our current perspective is that in the classroom situation of students working in groups on conceptually challenging mathematics, opportunities arise to fulfill some of the needs identified by Murray by pursuing motivating desires. The environmental press interacts with the individual's needs to impel *actions taken* toward satisfying the motivating desire.

There follows the current list of those structures, and how we conceptualize them:

Look How Smart I Am (LHSIA)

The motivating desire for this structure is to be seen as smart, primarily by classmates but perhaps also by the teacher (or by the student himself or herself, projected into the impressions of others). The student takes actions such as volunteering answers, possibly interrupting or dominating the discussion, or making suggestions that will be viewed by others as smart. The desired outcome is recognition of the student's mathematical capability, intelligence, or genius.

Let Me Teach You (LMTY)

The motivating desire for this structure is to teach another student (the “tutee”) something that the student knows that the other student does not know. The student takes actions such as listening to the ideas of the tutee, making suggestions, perhaps disagreeing with an answer or an idea suggested by the tutee, perhaps showing the tutee why he or she may be mistaken and how there might be a better way to approach the problem. The desired outcome is to have the student’s ideas considered by the tutee and accepted, so that the tutee reaches (and, possibly, understands) the correct solution.

Get The Job Done (GTJD)

The motivating desire for this structure is to make sure that all the required work is completed. The student takes actions such as writing answers to each part of the problem, or making sure his or her name is on the page, or enlisting others in completing the task as directed. The desired outcome is to have the teacher conclude that the student and/or the student’s group did all the required work satisfactorily.

Check This Out (CTO)

The motivating desire for this structure is to benefit from learning the math. The student takes actions such as working on the problem, thinking about possible solutions and trying to figure out the answer. The desired outcome is to learn something useful or to derive some other benefit as a consequence of working on the problem.

I’m Really Into This (IRIT)

The motivating desire for this structure is to understand the math. The student takes actions such as concentrating so intensely that he or she tunes out all distractions. The desired outcome is to truly understand and appreciate the mathematical ideas.

I’m Right, You’re Wrong (IRYW)

The motivating desire for this structure is to show another person that the student is right and the other person is wrong. The student takes actions such as arguing to support his ideas. The desired outcome is to have others recognize that his ideas were correct.

Stop The Class (STC)

The motivating desire for this structure is to get others to stop working on the math. The student takes disruptive actions such as making jokes or distracting others. The desired outcome is to disrupt the class activity.

Don’t Disrespect Me (DDM)

The motivating desire for this structure is to make sure nobody disrespects the student. The student takes actions such as vigorously defending himself or herself, or his or her ideas. The desired outcome is to be respected by classmates.

Value Me (VM)

The motivating desire for this structure is for the teacher to have a good opinion of the student. The student takes actions such as volunteering answers or having the teacher look at the student’s work. The desired outcome is that the teacher does in fact think highly of the student or the student’s work on this occasion.

Stay Out Of Trouble (SOOT)

The motivating desire for this structure is to find a way to stay out of trouble. The student takes actions such as not volunteering answers, refraining from speech that could be construed as criticizing the ideas of another student, or obeying the teacher or other students. The desired outcome is to avoid trouble.

Pseudo-Engagement (PE)

The motivating desire for this structure is to have people think the student is working on the problem even though he or she isn't. The student takes actions such as writing on the paper, or punching keys on the calculator, that have nothing to do with work on the problem. The desired outcome is to look good, with people thinking the student was working.

It's Not Fair (INF)

The motivating desire for this structure is to change something that the student thinks is unfair, redressing an inequity. The student takes actions such as pointing out to the teacher students who are not doing their share of the work, or insisting on a change. The desired outcome is that fairness is restored.

I Want Out (IWO)

The motivating desire for this structure is to avoid doing the math activity. The student takes actions such as distracting himself, asking to leave the room, telling the teacher he does not feel well and cannot concentrate on the problem, or volunteering for something else. The desired outcome is that the student does not work on the math.

Focus On Me (FOM)

The motivating desire for this structure is to be the center of attention. The student takes actions such as saying outrageous things, or being the "class clown." The desired outcome is that the student is noticed.

We created a new version of the questionnaire that contains questions intended to assess each of these 14 structures. Five sections of the new Inventory contain items designed to assess whether a given structure was active for a student during the class (see below).

A sixth section contains 22 adjectives used to assess emotional feelings. Eleven adjectives are positive (interested, respected, proud, successful, safe, excited, happy, satisfied, relieved, confident, curious). Eleven other adjectives are negative (unhappy, disappointed, worried. discouraged, angry, disrespected, bored, embarrassed, frustrated, afraid, confused.) These items are rated on a 5 point Likert Scale (Never, Hardly Ever, Sometimes, Often, Always).

Finally, there are two items following the list of adjectives, designed to assess self-perception of learning and participation. These are: 1. Compared to a typical math class I've been in this year, I learned _____ ; 2. Compared to a typical math class I've been in this year, I participated _____. These two items are rated on a 5 point Likert Scale (Much Less, A Little Less, About The Same, A Little More, Much More).

The five sections of the questionnaire used to assess whether an engagement structure was active are:

1. WHAT I WANTED TODAY IN MATH CLASS

This section assesses how much of the time the student experienced the motivating desire during the classroom session. Each item is rated on a 3 point Likert Scale (Never or hardly ever; Some of the time; Always or most of the time). Item 15 in this section assesses whether the student is answering in a deliberate manner, or possibly just going down the column and filling in the same answer for each question or answering randomly. Item 15 says, "I am reading each of these questions carefully."

2. WHAT I DID OR TRIED TO DO TODAY IN MATH CLASS

This section assesses how much of the time the student engaged in actions toward satisfying the motivating desire during the classroom session. Each item is rated on a 3 point Likert Scale (Never or hardly ever; Some of the time; Always or most of the time).

3. HOW MUCH WHAT I WANTED MATTERED TO ME TODAY IN MATH CLASS

This section assesses how important it was for the student to satisfy each motivating desire. Each item is rated on a 3 point Likert Scale (Very important for me; Somewhat important for me; Not very important for me).

4. HOW MUCH WHAT I DID MATTERED TO ME TODAY IN MATH CLASS

This section assesses how important it was for him to engage in actions to satisfy the motivating desire during the classroom session.. Each item is rated on a 3 point Likert Scale (Very important for me; Somewhat important for me; Not very important for me)

5. WHAT HAPPENED BY THE END OF MATH CLASS TODAY

This section assesses whether a given outcome occurred by the time class ended. The student says yes or no to each possible outcome. In addition to the questions assessing what happened as it related to the structures, there are two other items. One item assesses what happened with respect to self-identity: "I felt like a mathematical problem solver." The second item assesses what happened in terms of the meaningfulness of the activity for the student: "I accomplished something important." Finally, this section contains another item to ascertain whether the student is responding thoughtfully to the questionnaire: "I am reading each of these questions carefully."

Table 4 displays each of the 14 structures, and lists the questionnaire items that are used in each of the 5 sections of the Inventory to assess aspects of the structures. The order of questions mapping to structures is randomized within each section. Each question is followed by a number to indicate the order of the question in that section of the Inventory. Table 4 also includes our analysis of the correspondence with Murray's description of needs.

Table 4

Revised Questionnaire Items, Sections of the Questionnaire Where They are Located, and the Structures to Which They Map

Section	LHSIA	LMTY	GTJD	CTO	IRIT	IRYW	STC
1. WHAT I WANTED TODAY IN MATH CLASS (Frequency)	I wanted people to see that I am smart (11)	I wanted to teach another student something that I knew that this other student did not know (5)	I wanted to make sure that all the required work was completed (1)	I wanted to benefit from learning the math (4)	I wanted to understand the math (10)	I wanted to show another person that I was right and the other person was wrong (3)	I wanted to get others to stop working on the math. (8)
2. WHAT I DID OR TRIED TO DO TODAY IN MATH CLASS (Frequency)	I did things to impress people with my ideas about the problem (9)	I tried to help someone see how to do the math (4)	I worked on getting the answer to the problem (13)	I worked on learning the math (3)	I concentrated so deeply on my work that I tuned out things going on around me (10)	I disagreed openly with another person whose idea I thought was wrong (1)	I tried to find ways to disrupt the activity. (6)
3. HOW MUCH WHAT I WANTED MATTERED TO ME TODAY IN MATH CLASS (Importance)	It was ____ important for people to see that I am smart (8)	It was ____ important to me to teach another student something that I knew that this other student did not know (2)	It was ____ important to me to make sure that all the required work was completed (7)	It was ____ important to me to benefit from learning the math (14)	It was ____ important to me to understand the math (5)	It was ____ important to show another person that I was right and the other person was wrong (11)	It was ____ important to get others to stop working on the math (13)
4. HOW MUCH WHAT I DID MATTERED TO ME TODAY IN MATH CLASS (Importance)	It was ____ important to impress people with my ideas about the problem (9)	It was ____ important to help someone see how to do the math (2)	It was ____ important to work on getting the answer to the problem (5)	It was ____ important to work on learning the math (3)	It was ____ important to concentrate deeply on my work (11)	It was ____ important to disagree openly with another person whose idea I thought was wrong (14)	It was ____ important to find a way to get others to stop working on the math (6)
5. WHAT HAPPENED BY THE END OF MATH CLASS TODAY (Yes or No)	People saw how good I was at math (1)	I was able to help someone else see how to do the math. (5)	I got the answer to the problem (6)	I benefited from learning the math. (2)	I truly understood the math (10)	My ideas were recognized as being right. (4)	Because of me, people stopped working on the math. (12)
Needs underlying the motivating desire	Achievement: Murray p. 164 “To increase self-regard by the exercise of talent”	Nurturance: Murray p. 184 “To gratify the needs of…a mentally confused person.”	Deference: Murray p. 154 “To yield to the influence of an allied other (the teacher)”	Achievement: Murray p.164 “To master …ideas)	Understanding: Murray p. 224 “To represent in symbols the order of nature”	Dominance: Murray p. 152 “To convince O of the rightness of one’s opinion”	Dominance: Murray p. 152 “To influence the behavior of others by persuasion or command.”
Situational experience triggering the motivating desire	A potentially admiring audience; possibly (not necessarily) “rivals” for attaining high regard.	Noticing a person who does not understand aspects of the problem, while the person who notices understands more.	Instructions by the teacher to complete a task	An intellectually challenging problem	A problem that arouses the student’s interest	Encountering someone whose ideas seem inferior to own ideas	A dislike for the activities taking place in class

Section	DDM	VM	SOOT	PE	INF	IWO	FOM
1. WHAT I WANTED TODAY IN MATH CLASS (Frequency)	I wanted to make sure nobody disrespected me (6)	I wanted the teacher to have a good opinion of me. (9)	I wanted to find a way to stay out of trouble (2)	I wanted people to think I was working on the problem even though I wasn't. (7)	I wanted to change things that I thought were unfair. (12).	I wanted to get out of doing math. (14)	I wanted to be the center of attention. (13)
2. WHAT I DID OR TRIED TO DO TODAY IN MATH CLASS (Frequency)	I defended myself or my ideas. (11)	I tried to do the kind of things that would make the teacher think highly of me. (5)	I tried to avoid doing anything that might get me in trouble.(7)	I tried to do things to make people think I was working on the problem even though I wasn't. (12)	I spoke out about or tried to change something that I thought was unfair. (8)	.I looked for a way to avoid working on the problem. (14)	I tried to do things to make me the focus of attention. (2)
3. HOW MUCH WHAT I WANTED MATTERED TO ME TODAY IN MATH CLASS (Importance)	It was ____ important to make sure nobody disrespected me (6)	It was ____ important for the teacher to have a good opinion of me (12)	It was ____ important to find a way to stay out of trouble (3)	It was ____ important for people to think I was working on the problem even though I wasn't (1)	It was ____ important to change things that I thought were unfair (10)	It was ____ important to get out of doing the math (9)	It was ____ important to be the center of attention (4)
4. HOW MUCH WHAT I DID MATTERED TO ME TODAY IN MATH CLASS (Importance)	It was ____ important to defend myself when I was challenged (1)	It was ____ important to make the teacher think highly of me (10)	It was ____ important to do things to avoid getting in trouble (4)	It was ____ important to do things to make people think I was working on the problem even though I wasn't (8)	It was ____ important to change something that I thought was unfair (7)	It was ____ important to find a way to avoid working on the problem (12)	It was ____ important to find a way to be the center of attention (13)
5. WHAT HAPPENED BY THE END OF MATH CLASS TODAY (Yes or No)	People respected me (13)	The teacher thought highly of me. (7)	I was able to avoid trouble. (3)	People thought I was working on the math (9)	Things were fair. (11)	I avoided working on the math (8)	People paid a lot of attention to me. (14)
Needs underlying the motivating desire	Infavoidance: Murray p. 192 "To avoid conditions which may lead to belittlement"	Affiliation: Murray p. 174 "To win affection of a cathected other."	Harmavoidance: Murray p. 197 "To take precautionary measures."	Blameavoidance: Murray p. 187 "To avoid blame or rejection."	Succorance: Murray p. 182 "To have one's needs gratified by an allied other"	Autonomy: Murray p. 156 (to get free of restraints)	Exhibition: Murray p. 170 "To be seen and heard"
Situational experience triggering the motivating desire	Belittlement or ridicule for one's ideas	Uncertainty about whether the student's work is of high quality	Perception of others who would punish p for actions or inactions	Perception of punishment by teacher for lack of working on task.	An unfair set of circumstances	Being stifled	Being forgotten because others are doing things that draw attention to them.

Question numbers are in ( ) . LHSIA=Look How Smart I Am; **LMTY**=Let Me Teach You; **GTJD**=Get The Job Done; **CTO**=Check This Out; **IRIT**=I'm Really Into This; **IRYW**=I'm Right You're Wrong; **DDM**=Don't Disrespect Me; **VM**=Value Me; **SOOT**=Stay Out Of Trouble; **PE**=Pseudo-Engagement; **INF**=It's Not Fair; **IWO**=I Want Out; **FOM**=Focus on Me; **STC**=Stop The Class. Likert Scales: **Frequency**: Never or hardly ever; Some of the time; Always or most of the time. **Importance**: Very important for me; Somewhat important for me, Not very important for me. Order of questions assessing each structure is randomized as per the question numbers in parentheses for each of the 5 sections.

Our plan in the next phase is to assess whether any given structure was active for a given student by using the following rubric: We will consider a structure to have been active during a class session when: (1) the motivating desire is reported as experienced *at least* some of the time, and (2) that motivating desire is deemed to be *at least* somewhat important and, (3) the student reports engaging in action to satisfy the motivating desire *at least* some of the time.

Our conceptualization of engagement structures includes the idea that, as time passes, activity unfolds along pathways that contains "branch points." These branch points typically correspond to outcomes of actions taken to satisfy the motivating desire. Consider the following example beginning with activation of the LMTY structure. A student notices that a classmate seems to misunderstand the problem, or seems unable to figure out an aspect of the problem that the student herself is able to do. She has the desire to teach her classmate what she knows. She talks to the student, pointing out that he seems in need of information or help and that she is volunteering to provide it. This action is in the service of pursuing her motivating desire to help. At this juncture along the pathway, a branch point is reached. One possible outcome is for the tutee to accept the help, and continue working productively on the problem. An alternate outcome is for the tutee to reject the offer of help, and assume a negative attitude toward the tutor. By examining the checked outcomes marked on the Inventory, we can infer possibilities for branches that may have taken place along the pathways. We then examine the emotional feelings rated by the student. To continue the previous example, if the tutee accepted the offer of help, the tutor might indicate that she experienced pride, happiness, and/or satisfaction in conjunction with an active LMTY structure. In contrast, if the tutee rejected the offer of help, the tutor might indicate that she experienced unhappiness, disappointment, frustration, and/or disrespect in conjunction with an active LMTY structure. Thus the same structure can be associated with very different emotional feelings, as a function of the different pathways into which the structure may branch. In the present example, a negative outcome might lead to a different structure becoming active – DDM, or LHSIA.

In designing the revised questionnaire, we wanted to create the possibility of inferring up to 14 structures which we believe may influence mathematical engagement and learning. We realize that some of these structures may be difficult to distinguish from each other through questionnaire responses: for example CTO and IRIT, or LHSIA and IRYW. We also recognize that some the structures may be substructures of overarching superstructures. For example, the structure we have called **Value Me** has to do with a self-identity structure incorporating a wish to be important – to matter, to be valuable. That wish can be experienced in multiple realms: to matter to the teacher (**Value Me**), to matter to fellow students (to be considered smart, as in the **Look How Smart I Am** structure), to be noticed (as in the **Focus On Me** structure), or to be respected (as in the **DDM** structure). In other words, people have a strong desire to count for something – to be taken seriously, noticed, cared about, consulted, not dismissed or disrespected – and many of the engagement structures we have identified may develop as facets of that larger desire. Some may include tactics to increases the chances that this state of being held as important is achieved – for example, teaching another student something (a tactic in the **Let Me Teach You** structure), is also a way to achieve importance, to count, to be noticed and valuable and useful to others.

We envision that the next phase of our research will involve collecting data using the revised questionnaire, ascertaining which of these 14 structures emerge as measurable and distinguishable from each other with such an instrument, gathering evidence for further relationships among the structures, and exploring how they are in turn associated with mathematical learning and achievement.

Note 1:

The following administration instructions accompanied the prototype questionnaire:

Allow a minimum of 15 minutes. The amount of time might be more or less depending on reading level of students or other factors. We approximate about 5 minutes for handing out questionnaires and reading the introduction, and about 3 minutes per page.

You may wish to instruct them about filling out the heading: Full name; Date; It’s up to you about what students write for “Your Class”; If students worked in groups and you want the group members’ names, let them know that. We usually ask for first names only.

Mandatory introduction:

- Your responses are for the research team so your teacher will not see your answers. Please be as honest as possible. There is no right or wrong answer, just an honest answer.
- We’re interested in how you felt while working in math class and not how your neighbor or group felt. So write down the answers for yourself and no one else.
- Don’t consult with anyone else, talk to anyone or look at anyone else’s paper while you are filling out this questionnaire.
- If you have any questions about what a word or phrase means, please ask me. **(NOT TO BE READ: Don’t let the teacher answer questions.)**
- When you finish please place your paper face down on your desk.

During the questionnaire, please read the instructions at the top of each page and the first few questions of each section.

The top of the questionnaire contains these directions

THE WAY IT WAS FOR ME IN THIS CLASS TODAY

Please think about the time you spent in your group today. Write <u>one or two sentences</u> for each question.

1. What was the most memorable thing that happened while you were working in the group?
2. Was there any other thing that happened that stands out in your memory?
3. Did anything happen that made you feel especially good (for example, pleasant, happy)? If yes, what?

4. Did anything happen that made you feel especially bad (for example, unpleasant, unhappy)? If yes, what?
5. What stands out in your memory about something your teacher did or said today?

We will read the first few questions to you and then you will read them yourself. After each question, please indicate your answer. For each question, please circle one of the 3 answer choices.

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