When Students Disagree: Engagement and Understanding in an Urban Middle School Math Class

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We provide a description of the mathematical activity of students in a classroom, highlighting the different types of engagement that are enlisted, the mathematical understanding of two students (using the Pirie/Kieren model) and their role within their respective groups. Our goal is to better understand some of the many factors that can influence the way in which students react to criticism by their peers.

INTRODUCTION AND FRAMEWORK

In their research involving urban middle school students, Schorr, Epstein, Warner, and Arias (in press) and Epstein et al. (2007) note that students may be willing to abandon what they know are mathematical truths in order to avoid appearing weak or wrong in front of their peers. In particular, they discuss the case of Dana—a young student who defended a solution that she knew might be incorrect as another student, Shay, pointed out an error in her work in the presence of her group members. Rather than admit that she might have made a mistake, she vehemently defended her incorrect solution. In a follow-up interview (shortly after), Dana conceded that she becomes uncomfortable "…when people try to prove *me* wrong." Immediately prior to Shay's comment, Dana was interested in understanding the concept. After Shay's public criticism, her position changed to one that focused on avoiding looking foolish, or publicly losing face.

Dana's response is consistent with the findings of Dance (2002), Anderson (2000), and Devine, (1996), that students are often hypersensitive to situations in which their emotional safety, status, or wellbeing may be challenged. The main goal of this paper is to provide a "prequel" to the episode above between Dana and Shay (which is one part of a larger year long study involving several math classrooms), noting in particular the interplay of engagement and mathematical understanding, using the Pirie-Kieren (1994) model for the growth of mathematical understanding.

The Pirie-Kieren model (1994) provides a framework for analyzing the growth of understanding, via a number of layers through which students move both forward and backward. Pirie (1988) discussed the idea of using categories in characterizing the growth of understanding, observing understanding as a whole dynamic process and not as a single or multi-valued acquisition, nor as a linear combination of knowledge categories. Pirie & Kieren (1994) illustrate eight potential layers or distinct modes within the growth of understanding for a specific person, on any specific topic.

The middle school years can be a particularly stressful time for students. There is no doubt that most middle school students have a variety of issues and concerns that compete for their attention, and high among those are social issues involving things like peer group acceptance. It is therefore not surprising that actions instrumental to attaining acceptance, or avoiding confrontation, are likely to be allocated a large share of any student's attention. Indeed, in interviews during our study students often mentioned these issues.

Eccles and Midgley (1989) note that middle school students *need* an environment that provides a 'zone of comfort'. In our own work, we also note the importance of providing students with what we term "an emotionally safe environment". In such an environment, the students are free to question ideas, and openly discuss (mis)understandings without risk or fear of embarrassment or humiliation. Oftentimes, students in such an environment work in small groups and engage in mathematical discourse that includes efforts to prove and justify contentions to peers and the teacher. Implicit in this discourse is a socio-mathematical norm that permits and even encourages students to challenge the ideas put forth by fellow students (Cobb, Wood, & Yackel, 1993; Franke, Kazemi, & Battey, 2007). Of course, the teacher is of key importance in shaping the emotional safety of the classroom and the nature of the discourse that takes place. "How teachers and students talk with one another in the social context of the classroom is critical to what students learn about mathematics and about themselves as doers of mathematics" (Franke et al., 2007, p. 230). Nonetheless, given an emotionally safe environment, *and* a teacher who actively seeks to instill classroom norms that encourage productive and meaningful mathematical exploration and discourse, different students will engage with mathematical problems in different ways, ultimately impacting their overall understanding and interactions with each other. Further, despite the best intentions of the teacher, students may feel threatened or uncomfortable when their work is criticized by their peers. Our goal in this paper is to better understand the differing modes of engagement, how they impact learning, and how they unfold when students criticize each other's work.

To this end, we have identified several different types of *student engagement structures* (see Goldin, Epstein, & Schorr, 2007). Certain structures contribute directly to mathematical engagement, while others, at times, impede it. We see most or all of these structures as present within individuals and becoming operative under given sets of circumstances. We have identified at least seven engagement structures ranging from extreme engagement, akin to what Csikszentmihalyi (1990) describes as "flow" (complete immersion in a task or activity), to complete disengagement, where students do what Kohl (1994) describes as "not learning". For the purposes of this report, we focus on three types of engagement structures: A. *Check This Out*: In this structure, the student is highly engaged in the task, often to the exclusion of other events that may be occurring within the context of the group or the classroom; B. *Get The Job Done*: This structure involves a person's sense of obligation to fulfill his part of a work "contract." The student is much more aware of anything that helps or hinders progress toward that goal; and, C. *Don't Disrespect Me*: This structure involves the person's experience of a perceived challenge or threat to his or her wellbeing, status, dignity, or safety. Resistance to the challenge raises the conflict to a level above that of the original mathematical task. The need to maintain “face” supersedes the mathematical issues.

Different structures vary in the degree of engagement that they recruit. The greatest engagement of the structures described above occurs in the “Check This Out” structure. Less intense engagement is seen in the "Get The Job Done" structure. As part of our analysis, we also consider what psychologists may refer to as “figure” and “ground” (popularized by the Danish psychologist Rubin, 2001). Figure, as we use it here, refers to the primary focus of attention, whereas ground refers to that which is present in the background. In our work, we have found that, at times, the mathematics may be figure and other aspects of the context may be ground, and vice versa.

METHODS

Subjects: The 8^{th} grade classroom that is the focus of this research consisted of 20 students, 93% African American and 7% Hispanic. The school, classified as “low income,” is in the largest city in the state of New Jersey. This class was homogeneous and designated as a low ability class (lowest in the grade level). The teacher encouraged what we describe above as an “emotionally safe” learning environment for students, a necessary condition for inclusion in the larger study.

Procedure: In the larger study, classes were observed in each of four “cycles,” with each cycle spanning a period of two consecutive days. The first cycle occurred approximately one month into the school year and subsequent cycles occurred later on. Prior to the start of a cycle, an interview was conducted with the teacher to ascertain her plans for the lesson and what she expected to happen. A follow-up interview (using a stimulated recall protocol) with the teacher and several students took place after each cycle (for more details see Epstein et al., 2007). Classroom interactions for each of the classes were videotaped using three separate cameras and all student and teacher interviews were videotaped using one camera. Transcripts were created from videotapes and student work was collected.

Analysis: A team of researchers from the fields of mathematics education, social psychology, mathematics, and cognitive science reviewed and analyzed the results. All videos were viewed through four distinct, yet overlapping lenses: the mathematical (cognitive) lens; the affective lens, particularly with regards to engagement; teacher interventions (including actions, behaviors, etc.); and social interactions. In all cases, the structures that we have identified were created <u>after</u> observing the data, rather than ahead of time.

For this paper, we analyze data from one class during cycle one over the span of two days (about 43 minutes each day). The students were working in groups of three to five (groups formed according to the typical seating arrangements-no roles or assigned tasks were given to any students) on the following task:

Farmer Joe has a cow named Bessie. He bought 100 feet of fencing. He needs you to *help him create a rectangular fenced in space with the maximum area for Bessie to graze. Bullet 1: Draw a diagram with the length and the width to show the maximum area. Bullet 2: Explain how you found the maximum area. Bullet 3: How many poles would you have for this area if you need 1 pole every 5 feet?*

RESULTS AND DISCUSSION

We share results by comparing and contrasting Dana and Shay with respect to engagement structures and the role they assumed within their respective groups. We then offer a more complete description involving their respective mathematical behaviors.

Student Name	Engagement Structure	Role Within Group
Shay (Day 1: beginning to almost end of class)	Check This Out	Worked alone for most of session; occasionally shared ideas with group mates & teacher; at times, asked group mates to help supply him with paper & a calculator.
Shay (Day 1: last 5 minutes & 1st half of Day 2)	Check This Out	Took on role of leader, assigned roles to group mates; asked group mates to supply him with tools (yardstick, scrap paper); recorded group's solution on chart paper to share with the class.
Shay (middle of Day 2)	Check This Out	Shared ideas with teacher and group mates; critiqued Dana's group's work.
Dana (1st half of Day 1)	Check This Out	Asked questions about mathematical ideas; made requests for tools (calculator & scrap paper).
Dana (last half of Day 1 & 1st half of Day 2)	Get The Job Done	Monitored every group member's progress to make sure they were all on task; made requests for tools (i.e. chart paper); recorded group's solution on chart paper to share with the class.
Dana (middle of Day 2)	Check This Out	Shared mathematical ideas; asked questions about Shay's group's solution.

Table 1: Engagement structures and roles within the group.

Shay: Shay, a young male student who is described by his teacher as being a bright and “street wise” student, began investigating the task alone while the other three members of his group worked together, spending a considerable amount of time talking about other non-mathematical issues (socially related). Thirteen minutes later, Shay announced to his group, in an apparent breakthrough, “You could do a lot of ‘em, it could be like…wait…” Realizing that there could be many rectangles with a perimeter of 100 feet, he encouraged his group members to investigate some possibilities while he continued to quietly work, primarily alone, for most of the class. For the most part, he interacted with the others only when the teacher asked questions and/or when the teacher encouraged them to share ideas with each other.

During this time, Shay expressed some difficulty in figuring out the length and width of potential rectangles. We suggest that he was functioning in the *image making layer* of the Pirie & Kieren model (2004) in that he still needed to draw specific rectangles in order to create an image of what each potential solution could be, but was still tied to the action of drawing in order to figure out the length of the sides in any one specific case. As he continued to work, he began to develop strategies for building rectangles with a perimeter of 100. He developed a method for finding the dimensions of rectangles, which involved finding the width for a specific length. He added the side length to itself (for the length of two sides), subtracted the sum from 100, and divided the difference by two (to find the width). At this point, it appeared as though he had reached a *don't need* boundary and had an image of how he could construct rectangles with a set perimeter of 100. He was, at this time, working in the *image having layer* because he was no longer tied to the action of drawing the rectangle.

At the very end of the first session, Shay realized that the other group members were still not sure about how to find the area of these rectangles. He also realized that it would take a long time to construct all *possible* rectangles with a perimeter of 100 feet (using only whole numbers). He stated, "you can keep going but it can go all day". He now, for what appeared to be very practical reasons, assumed the role of leader in his group, noting, "Move, I know what to do, look…what number, look… we gonna start, I'm gonna do the lower numbers, you do the higher numbers…" The other three members of his group began working independently on the different rectangles that he assigned them.

As he and the members of his group continued to work on the problem, Shay realized that he could find every (whole number) rectangle by increasing the length by one and decreasing the width by one. He now had an image of the construction of these rectangles and was no longer tied to the action of drawing each one to figure out the length and width. He continued with this strategy until he came to the conclusion that the rectangle with the maximum area had dimensions of 26 by 24 (because he didn't consider the 25 by 25 square to be a rectangle). At this point, Shay also became invested in seeing to it that his group mates understood his method.

Dana: Dana, a young female student, described by her teacher as being popular and eager to please, but also "tough", worked on the task with 3-4 other students. In the beginning of the session, Dana attempted to understand the task by asking questions and trying to figure out the meaning of area. Despite some misconceptions (especially relating to area) Dana spontaneously took on the role of group leader, often telling her group mates what to do and when to do it. Her overall approach was to find the area of a rectangle that appeared to meet the conditions of the problem. After finding one such rectangle, Dana directed the group to consider another part of the problem task (related to the number of poles).

After a short time, Dana told the group, "Yes. We need eight poles, so for the second bullet umm…two poles…two poles…" "So that's…stop (to another group member) so that's something that's asking for bullet one [bullet refers to the different parts of the problem task], and bullet two, and bullet three. So, is everyone caught up yet (addressing the other four members of her group)? Ya got bullet three (as she monitored their work)?" Dana was concerned when someone in her group was off task—perhaps because it might get in the way of completing all of the parts of the problem.

We suggest that Dana did not have an image of how to create different rectangles with a constant perimeter when she finished the task. She used, as her final solution, the first rectangle (40 by 10) she constructed that had a perimeter of 100. She didn't explore any other possibilities. She noted that in order to find the maximum area, you needed to "multiply length times width" without actually doing so. She never progressed past the *image making layer*, and unlike Shay, didn't reach a *don't need boundary* (where she was no longer tied to the action of drawing the rectangle).

Engagement structures: Based upon supporting evidence from the data, we conclude that Shay spent most (if not all) of his time in the "Check This Out" structure, where the mathematics was figure, and the other aspects of the classroom context (social, for example) were ground. It was only after he was sure of his method that he monitored the progress of his group, and even then, the mathematics was central.

Dana, on the other hand, monitored the progress of her group throughout the entire session, trying to enlist their attention when it seemed to fade. While she was open to the ideas of her peers, she remained the sole arbiter of what ideas would be pursued and what ideas would not. We suggest that for Dana, the primary engagement structure was "Get The Job Done" since she appeared to be mainly interested in being sure that she had answered all parts of the question rather than formulating a more complete solution for each. In Dana's case, the mathematics was ground, and monitoring her group's activity was figure.

Critiquing each other's work: In both cases, Shay and Dana recorded the solutions for their respective groups, and therefore had considerable ownership of the work. After all of the groups had recorded their solutions, the teacher asked the students to walk around and review the work of the other groups, noting on small pieces of paper any questions that they had about another group's work.

During this time, Dana noticed a major difference in the way that she had solved the problem and the way that Shay's group had solved it, and she raised questions about Shay's solution to members of her group, trying intently to understand it:

Dana But I don't know how they got this, how they got maximum area…(inaudible) … I don't get that But I want to know how do they get the answer. I want to know how they get this…they got this. I don't understand how they got this.
Von You add it up. You do it and see what you get.
Dana I'm talking about this, and why did they do this (referring to the dimensions of the largest rectangle)?
Dana Look, they said length times the width here; they multiplied the length and the

width.

Von Could we just agree on something?

Will No, just write, "We don't understand how they got the maximum area." Write that.

Von Write that? "We don't understand how y'all got your maximum area" (referring to Shay's group).

Dana Ok, yes I do. I got it. *They are right*. 'Cause they said they got 26, they multiplied 26, so they multiplied this one for, um…Could say good job because they did. First, I didn't understand it.

Analysis of the data (interview, classroom, field notes) leads us to believe that at this point in time Dana moved into the "Check This Out" structure, as she tried intently to understand Shay's group's solution, and no longer needed to monitor her group. It now appeared that for Dana, the mathematics became figure. Shay, on the other hand, remained in the "Check This Out" structure as he openly explored each group's solution. However, as reported in Schorr et al. (in press) and Epstein et al. (2007), Dana went into the "Don't Disrespect Me" structure when she heard Shay criticize her work. She was adamant in her response to Shay, and acted defensively regarding the accuracy of her solution.

CONCLUSIONS

Our analysis is intended to highlight several points. First, we note that within the context of a single classroom, different types of engagement structures can simultaneously be enlisted. Further, an individual may move into and out of several different types of structures depending on the circumstance.

In Dana's case, one might speculate that she had much to lose by admitting that she had, potentially, led her group down an incorrect (or at least incomplete) path. Further, because she had a somewhat limited understanding of the math, she may have felt that she could not explain her ideas sufficiently well to others, if challenged. Finally, she knew, to some extent that Shay's ideas *were* correct, and hers might have some flaws. Taken together, these factors contributed to Dana's reaction to Shay.

By considering the confluence of many factors (at the very least, mathematical understanding, role within group, and perceived ability to defend one's work), we can better understand how students may respond to their peer's criticism, even in the context of an emotionally safe environment.

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